Minor Losses (Local)

Pipe entrance or exit
Sudden expansion or contraction
Bends, elbows, tees, and other fittings
Valves, open or partially closed
Gradual expansions or contractions
Total Head Loss

\[ \text{total loss} = H_1 - H_2 \quad h_\lambda = h_f + h_m \]

friction loss: \( h_f = f \times (L/D) \times (V^2/2g) \)

minor loss: \( h_m = K_L \times (V^2/2g) \)

\( K_L \) is the loss coefficient

For each pipe segment (i.e. reaches along which pipe diameter remains constant) there may be several minor losses.
Pipe section with valve:

Pipe section without valve:

\[ \Delta P_L = (P_1 - P_2)_\text{valve} - (P_1 - P_2)_\text{pipe} \]
Sharp-edged inlet
$K_L = 0.50$

Well-rounded inlet
$K_L = 0.03$

Recirculating flow

Vena contracta

Separation flow

Pressure head converted to velocity head

Lost velocity head

Remaining velocity head

Remaining pressure head

Total head

$\frac{V_1^2}{2g}$

$\frac{V_2^2}{2g}$

$\frac{P_0}{\rho g}$

$\frac{P_1}{\rho g}$

$\frac{P_2}{\rho g}$
Flow in an elbow.
Flanged elbow
$K_L = 0.3$

Sharp turn
$K_L = 1.1$
Expanding Flows
Experiments show that the pressure at 3-3 is equal to P1.

Ignoring the friction force:

\[
Q = V_1 A_1 = V_2 A_2
\]
Expanding flow

Continuity eq.: \( Q = V_1A_1 = V_2A_2 \)

Momentum eq. In x-direction: \( \gamma \frac{(p_1 - p_2)A_2}{g} = \frac{1}{\gamma} (V_2^2 - V_1V_2) \)

Energy eq.: \( z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_m \)

\( h_m = \frac{p_1 - p_2}{\gamma} + \frac{V_1^2 - V_2^2}{2g} \)

\( h_m = \frac{(V_2 - V_1)^2}{2g} \), \( V_2 = \frac{V_1A_1}{A_2} = \frac{V_1D_1^2}{D_2^2} \)

\( h_m = \left( \frac{D_1}{D_2} - 1 \right)^2 \frac{V_1^2}{2g} \), \( h_m = K_m \frac{V_1^2}{2g} \)
Determination of Local Loss ($h_m$):

$h_m = K_m \frac{V^2}{2g}$

<table>
<thead>
<tr>
<th>Type of fitting</th>
<th>Screwed</th>
<th></th>
<th>Flanged</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.5 cm</td>
<td>5 in.</td>
<td>10 cm</td>
<td>5 cm</td>
</tr>
<tr>
<td>Globe valve (fully open)</td>
<td>8.2</td>
<td>6.9</td>
<td>5.7</td>
<td>8.5</td>
</tr>
<tr>
<td>(half open)</td>
<td>20</td>
<td>17</td>
<td>14</td>
<td>21</td>
</tr>
<tr>
<td>(one-quarter open)</td>
<td>57</td>
<td>48</td>
<td>40</td>
<td>60</td>
</tr>
<tr>
<td>Angle valve (fully open)</td>
<td>4.7</td>
<td>2.0</td>
<td>1.0</td>
<td>2.4</td>
</tr>
<tr>
<td>Swing check valve (fully open)</td>
<td>2.9</td>
<td>2.1</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Gate valve (fully open)</td>
<td>0.24</td>
<td>0.16</td>
<td>0.11</td>
<td>0.25</td>
</tr>
<tr>
<td>Return bend</td>
<td>1.3</td>
<td>0.95</td>
<td>0.64</td>
<td>0.55</td>
</tr>
<tr>
<td>Tee (branch)</td>
<td>1.8</td>
<td>1.4</td>
<td>1.1</td>
<td>0.80</td>
</tr>
<tr>
<td>Tee (line)</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.19</td>
</tr>
<tr>
<td>Standard elbow</td>
<td>1.5</td>
<td>0.95</td>
<td>0.64</td>
<td>0.39</td>
</tr>
<tr>
<td>Long sweep elbow</td>
<td>0.72</td>
<td>0.41</td>
<td>0.23</td>
<td>0.30</td>
</tr>
<tr>
<td>45° elbow</td>
<td>0.32</td>
<td>0.30</td>
<td>0.29</td>
<td></td>
</tr>
</tbody>
</table>

- Square-edged entrance: 0.5
- Reentrant entrance: 0.8
- Well-rounded entrance: 0.03
- Pipe exit: Area ratio 1.0
  - Sudden contraction: 2:1, 0.25; 5:1, 0.41; 10:1, 0.46
  - Area ratio $A/A_0$: 1.5:1, 0.85; 2:1, 3.4; 4:1, 29; $\geq 6:1$, $2.78\left(\frac{A}{A_0} - 0.6\right)^2$
- Sudden enlargement: $1 - \frac{A_1}{A_2}$
  - 90° miter bend (without vanes): 1.1
  - (with vanes): 0.2
- General contraction: (30° included angle), (70° included angle) 0.02, 0.07

*Values for other geometries can be found in *Technical Paper 410*, The Crane Company, 1957.

*Based on exit velocity $V_2$.
*Based on entrance velocity $V_1$. 
Friction loss for non-circular conduits

Circular

\[
R_h = \frac{A}{P}
\]

\[
D_h = D
\]

\[
h_f = f \frac{L V^2}{D 2g}
\]

\[
Re = \frac{D V}{v}
\]

\[
f = f\left(Re, \frac{\varepsilon}{D}\right)
\]

Non-circular

\[
R_h = \frac{A}{P}
\]

\[
D_h = 4R_h
\]

\[
h_f = f \frac{L V^2}{D_h 2g}
\]

\[
Re = \frac{D_h V}{v}
\]

\[
f = f\left(Re, \frac{\varepsilon}{D_h}\right)
\]
Friction loss for non-circular conduits

for $P_2 = P_1$

$
A_2 < A_1
$

$D_h < D$

$V_2 > V_1$

$\frac{\varepsilon}{D_h} > \frac{\varepsilon}{D}$ and $R_{e2} \approx R_{e1} \Rightarrow f_2 > f_1$

$h_{f2} > h_{f1}$
Hydraulic Diameter

Circular tube:
\[ D_h = \frac{4(\pi D^2/4)}{\pi D} = D \]

Square duct:
\[ D_h = \frac{4a^2}{4a} = a \]

Rectangular duct:
\[ D_h = \frac{4ab}{2(a + b)} = \frac{2ab}{a + b} \]
Example

Water flows from the basement to the second floor through the 2 cm diameter copper pipe (a drawn tubing) at a rate of \( Q = 0.8 \text{ lt/s} \) and exits through a faucet of diameter 1.3 cm as shown in figure. Determine the pressure at point 1 if:

a) viscous effects are neglected,

b) the only losses included are major losses

c) all losses are included

Given:

\[
egin{align*}
Q &= 0.8 \text{ lt/s} \\
D_p &= 2 \text{ cm} = 0.02 \text{ m} \\
D_f &= 1.3 \text{ cm} = 0.013 \text{ m} \\
s &= 1.5 \times 10^{-6} \text{ m} \\
\rho &= 1000 \text{ kg/m}^3 \\
v &= 1.13 \times 10^{-6} \text{ m}^2/\text{s}
\end{align*}
\]
Q = 0.8 lt/s
Pipe diameter = 2cm
Viscosity = 1.13 * 10^{-6} m^2/sn
Pressure at point 1?????

\[ V = \frac{Q}{A} = \frac{0.0008}{3.1416 \times 10^{-4}} = 2.546 \text{ m/s} \]

\[ \text{Re} = \frac{VD}{v} = \frac{2.546 \times 0.02}{1.13 \times 10^{-6}} = 45062 = 4.5 \times 10^4 \quad \text{flow is turbulent} \]

a) Energy equation between (1) and (9)

\[ H_1 = H_9 \]

\[ \frac{V_1^2}{2g} + \frac{P_1}{\gamma} + z_1 = \frac{V_9^2}{2g} + \frac{P_9}{\gamma} + z_9 \]

\[ V_1 = V = 2.546 \text{ m/s} , \quad V_9 = \frac{Q}{A_9} = 6.027 \text{ m/s} \]

\[ P_1 = \gamma \left[ \frac{V_9^2 - V_1^2}{2g} + z_9 \right] \]

\[ P_1 = 9810 \left[ \frac{6.027^2 - 2.546^2}{2 \times 9.81} + 8.12 \right] = 94578.5 \text{ Pa} = 94.6 \text{ kPa} \]

\[ \frac{P_1}{\gamma} = 9.64 \text{ m} \quad \text{Compare with} \quad z = 8.12 \text{ m} \]
b) \( H_1 - h_f = H_9 \) \[ P_1 = \gamma \left[ \frac{V_9^2 - V_1^2}{2g} + z_9 + f \frac{L}{D} \frac{V^2}{2g} \right] \]

drawn tubing, \( \epsilon = 0.00016 \text{ cm} \) \[ \epsilon/D = 8.10^{-5}, \ \text{Re} = 45062 \]

\( L_{\text{total}} = 4.6 + 3 \times 6 = 22.6 \text{ m} \)

\[ P_1 = 9810 \left[ \frac{6.027^2 - 2.546^2}{2 \times 9.81} + 8.12 + 0.0215 \frac{22.6}{0.02} \frac{2.546^2}{2 \times 9.81} \right] \]

\[ P_1 = 173320 \text{ Pa} = 173.3 \text{ kPa} \]

\[ \frac{P_1}{\gamma} = 17.67 \text{ m} \]
Moody diagram. (From L.F. Moody, *Trans. ASME*, Vol.66,1944.) (Note: If $e/D = 0.01$ and $Re = 10^4$, the dot locates $f = 0.043$.)

- Laminar flow
- Critical zone
- Transition zone
- Completely turbulent regime

<table>
<thead>
<tr>
<th>Material</th>
<th>$e$ (in)</th>
<th>$e$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riveted steel</td>
<td>0.01</td>
<td>3</td>
</tr>
<tr>
<td>Concrete</td>
<td>0.001</td>
<td>0.3-3</td>
</tr>
<tr>
<td>Wood</td>
<td>0.001</td>
<td>0.3</td>
</tr>
<tr>
<td>Cast iron</td>
<td>0.0065</td>
<td>0.26</td>
</tr>
<tr>
<td>Galvanized iron</td>
<td>0.0065</td>
<td>0.15</td>
</tr>
<tr>
<td>Wrought iron</td>
<td>0.0015</td>
<td>0.046</td>
</tr>
<tr>
<td>Drawn tubing</td>
<td>0.000005</td>
<td>0.0015</td>
</tr>
</tbody>
</table>

Smooth pipe

(c) 2002 Wadsworth Group/Thomson Learning
c) \[ H_1 - h_L - \sum_{i=2}^{8} K_i \frac{V_i^2}{2g} = H_9 \]

\[ P_1 = \gamma \left[ \frac{V_9^2 - V_1^2}{2g} + z_9 + f \frac{L}{D} \frac{V^2}{2g} + \sum K_i \frac{V_i^2}{2g} \right] \]

\[ \sum K_i = 1.5 + 1.5 + 0.4 + 0.4 + 1.5 + 10 + 2 = 17.3 \]

\[ P_1 = 9810 \left[ \frac{6.027^2 - 2.546^2}{2 \times 9.81} + 8.12 + 0.0215 \frac{22.6}{0.02} \frac{2.546^2}{2 \times 9.81} + 17.3 \frac{2.546^2}{2 \times 9.81} \right] \]

\[ P_1 = 229390.3 \text{Pa} = 229.4 \text{kPa} \quad \frac{P_1}{\gamma} = 23.38 \text{m} \]
This pressure drop (229.4 kPa) calculated by including all losses should be the most realistic answer of the three cases considered.

Comparison:

<table>
<thead>
<tr>
<th></th>
<th>$h_i=0$</th>
<th>$h_i=h_f$</th>
<th>$h_i=h_f+h_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$ (kPa)</td>
<td>94.6</td>
<td>173.3</td>
<td>229.4</td>
</tr>
<tr>
<td>$P_1/\gamma$ (m)</td>
<td>9.64</td>
<td>17.67</td>
<td>23.38</td>
</tr>
</tbody>
</table>

Given:
- $Q = 0.8 \text{ ft/s}$
- $D_F = 2 \text{ cm} = 0.02 \text{ m}$
- $D_r = 1.3 \text{ cm} = 0.013 \text{ m}$
- $s = 1.5 \times 10^{-4} \text{ m}$
- $\rho = 1000 \text{ kg/m}^3$
- $\nu = 1.13 \times 10^{-6} \text{ m}^2/\text{s}$
Piping Networks

• Two general types of networks
  – Pipes in series
    • Volume flow rate is constant
    • Head loss is the summation of components
  – Pipes in parallel
    • Volume flow rate is the sum of the components
    • Pressure loss across all branches is the same
Pipes in Series

\[ Q_A = Q_B \]

\[ h_{L, 1-2} = h_{L, A} + h_{L, B} \]
Pipes in Series

- For pipes connected in series:
  - \( Q = \text{constant} \quad Q_A = Q_1 = Q_2 = \cdots = Q_n = Q_B \)

- But the head loss is additive:
  - \( h_{\ell} = \sum h_{fi} + \sum h_{mi}, \quad i = 1, 2, \ldots, n \)

Where \( h_{fi} \) is the frictional loss in i-th pipe, and \( h_{mi} \) is the local loss in i-th pipe.
Example

- Given a three-pipe series system as shown below. The total pressure drop is $p_A - p_B = 150$ kPa, and the elevation drop is $z_A - z_B = 5$ m. The pipe data are

<table>
<thead>
<tr>
<th>pipe</th>
<th>L (m)</th>
<th>D (cm)</th>
<th>$\varepsilon$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>8</td>
<td>0.24</td>
</tr>
<tr>
<td>2</td>
<td>150</td>
<td>6</td>
<td>0.12</td>
</tr>
<tr>
<td>3</td>
<td>80</td>
<td>4</td>
<td>0.20</td>
</tr>
</tbody>
</table>

- The fluid is water ($\rho = 1000$ kg/m$^3$, $\nu = 1.02 \times 10^{-6}$ m$^2$/s). Calculate the flow rate $Q$ (m$^3$/hr). Neglect minor losses.
\[ H_A = H_B + \sum h_f + \sum h_m \]

\[
\frac{V_1^2}{2g} + \frac{V_3^2}{2g} + z_A = \frac{V_2^2}{2g} + z_B + f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} + f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g} + f_3 \frac{L_3}{D_3} \frac{V_3^2}{2g}
\]

\[
\left( \frac{p_A - p_B}{\rho} \right) + (z_A - z_B) = \left( f_1 \frac{L_1}{D_1} - 1 \right) \frac{V_1^2}{2g} + f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g} + \left( f_3 \frac{L_3}{D_3} + 1 \right) \frac{V_3^2}{2g}
\]
\[
\left( \frac{p_A - p_B}{\gamma} \right) + (z_A - z_B) = \left( f_1 \frac{L_1}{D_1} - 1 \right) \frac{V_1^2}{2g} + f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g} + \left( f_3 \frac{L_3}{D_3} + 1 \right) \frac{V_3^2}{2g}
\]

from continuity: \( V_1 A_1 = V_2 A_2 = V_3 A_3 \)

\[
A_1 = \left( \frac{8}{6} \right)^2 A_2 = \left( \frac{8}{4} \right)^2 A_3 \quad \text{and} \quad A_1 = 1.78 A_2 = 4 A_3
\]

\[
V_2 = 1.78 V_1 \quad \text{and} \quad V_3 = 4 V_1
\]

\[
\left( \frac{p_A - p_B}{\gamma} \right) + (z_A - z_B) = \left( f_1 \frac{L_1}{D_1} - 1 \right) \frac{V_1^2}{2g} + (1.78)^2 f_2 \frac{L_2}{D_2} \frac{V_1^2}{2g} \\
+ (4)^2 \left( f_3 \frac{L_3}{D_3} + 1 \right) \frac{V_1^2}{2g}
\]

\[
20.3 = (1250f_1 + 7920f_2 + 32000f_3 + 15) \frac{V_1^2}{2g}
\]
\[ V_2 = 1.78 \, V_1 \quad V_3 = 4.0 \, V_1 \]

\[ 20.3 = (1250 \, f_1 + 7920 \, f_2 + 32000 \, f_3 + 15) \, V_1^2 / 2g \]

Velocities and \( f \) not known. Thus, assume rough flow

<table>
<thead>
<tr>
<th>Pipe</th>
<th>( \varepsilon ) (mm)</th>
<th>D (cm)</th>
<th>( \varepsilon / D )</th>
<th>( f_i )</th>
<th>( V ) (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.24</td>
<td>8</td>
<td>0.003</td>
<td>0.026</td>
<td>0.579</td>
</tr>
<tr>
<td>2</td>
<td>0.12</td>
<td>6</td>
<td>0.002</td>
<td>0.023</td>
<td>1.030</td>
</tr>
<tr>
<td>3</td>
<td>0.20</td>
<td>4</td>
<td>0.005</td>
<td>0.030</td>
<td>2.314</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pipe</th>
<th>( \varepsilon ) (mm)</th>
<th>D (cm)</th>
<th>( \varepsilon / D )</th>
<th>( f_i )</th>
<th>( V ) (m/sec)</th>
<th>( Re )</th>
<th>( f_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.24</td>
<td>8</td>
<td>0.003</td>
<td>0.026</td>
<td>0.579</td>
<td>45381</td>
<td>0.029</td>
</tr>
<tr>
<td>2</td>
<td>0.12</td>
<td>6</td>
<td>0.002</td>
<td>0.023</td>
<td>1.030</td>
<td>60584</td>
<td>0.027</td>
</tr>
<tr>
<td>3</td>
<td>0.20</td>
<td>4</td>
<td>0.005</td>
<td>0.030</td>
<td>2.314</td>
<td>90762</td>
<td>0.031</td>
</tr>
</tbody>
</table>
\[ V_2 = 1.78 \, V_1 \quad V_3 = 4.0 \, V_1 \]
\[ 20.3 = (1250 \, f_1 + 7920 \, f_2 + 32000 \, f_3 + 15) \, V_1^2 / 2g \]

<table>
<thead>
<tr>
<th>Pipe</th>
<th>( \varepsilon )</th>
<th>D (mm)</th>
<th>( \varepsilon/D )</th>
<th>( f_i )</th>
<th>( V ) (m/sec)</th>
<th>( Re )</th>
<th>( f_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.24</td>
<td>8</td>
<td>0.003</td>
<td>0.029</td>
<td>0.563</td>
<td>44147</td>
<td>0.029</td>
</tr>
<tr>
<td>2</td>
<td>0.12</td>
<td>6</td>
<td>0.002</td>
<td>0.027</td>
<td>1.002</td>
<td>58937</td>
<td>0.027</td>
</tr>
<tr>
<td>3</td>
<td>0.20</td>
<td>4</td>
<td>0.005</td>
<td>0.031</td>
<td>2.252</td>
<td>88295</td>
<td>0.031</td>
</tr>
</tbody>
</table>

Close enough. Thus,

\[ V_1 = 0.563 \, \text{m/sec} \text{ or } Q = 0.0028 \, \text{m}^3 /\text{sec} \]
Equivalent Pipe Concept

For pipes in series:

- Consider two pipes connected in series:

\[A, L_A, D_A, \varepsilon_A\] and \[B, L_B, D_B, \varepsilon_B\]
• We want to replace these two pipes with an equivalent pipe C:

\[ h_{f12} = h_{fA} + h_{fB} = h_{fC} \] (1)

• and

\[ Q_A = Q_B = Q_C \] (2)
• The Darcy-Weissbach Equation:

\[ h_f = f \frac{L}{D} \frac{V^2}{2g} \]  and inserting \( V = \frac{4Q}{\pi D^2} \):

\[ h_f = 8f \frac{L}{D^5} \frac{Q^2}{g\pi^2} \]

• Therefore Eq.(1) can be written as:

\[ 8f_A \frac{L_A}{D_A^5} \frac{Q_A^2}{g\pi^2} + 8f_B \frac{L_B}{D_B^5} \frac{Q_B^2}{g\pi^2} = 8f_C \frac{L_C}{D_C^5} \frac{Q_C^2}{g\pi^2} \]  or

\[ f_A \frac{L_A}{D_A^5} + f_B \frac{L_B}{D_B^5} = f_C \frac{L_C}{D_C^5} = f_{eq} \frac{L_{eq}}{D_{eq}^5} \]
• If \( f_A = f_B = f_C \), then

\[
\frac{L_{eq}}{D_{eq}^5} = \frac{L_A}{D_A^5} + \frac{L_B}{D_B^5}
\]

• Generalizing for \( n \) pipes connected in series

\[
\frac{L_{eq}}{D_{eq}^5} = \frac{\sum_{i=1}^{n} L_i}{\sum_{i=1}^{n} D_i^5}
\]

• We choose either \( D_{eq} \), or \( L_{eq} \), then compute the other from the equation.
Example

- Given a three-pipe series system as shown below. The total pressure drop is $p_A - p_B = 150$ kPa, and the elevation drop is $z_A - z_B = 5$ m. The pipe data are

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<tr>
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<td>80</td>
<td>4</td>
<td>0.20</td>
</tr>
</tbody>
</table>

- The fluid is water ($\rho = 1000$ kg/m$^3$, $\nu = 1.02 \times 10^{-6}$ m$^2$/s). Calculate the flow rate $Q$ (m$^3$/hr). Neglect minor losses.
choose either $D_{eq}$, or $L_{eq}$, then compute

the other from the equation

$$\frac{L_{eq}}{D_{eq}^5} = \frac{\sum_{i=1}^{n} L_i}{\sum_{i=1}^{n} D_i^5}$$

<table>
<thead>
<tr>
<th>Pipe</th>
<th>$\varepsilon$ (mm)</th>
<th>D (cm)</th>
<th>$\varepsilon/D$</th>
<th>L (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.24</td>
<td>8</td>
<td>0.003</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>0.12</td>
<td>6</td>
<td>0.002</td>
<td>150</td>
</tr>
<tr>
<td>3</td>
<td>0.20</td>
<td>4</td>
<td>0.005</td>
<td>80</td>
</tr>
</tbody>
</table>

Let’s use $L_{eq} = 330 \text{m}$

$$\frac{330}{D^5} = \frac{100}{0.08^5} + \frac{150}{0.06^5} + \frac{80}{0.04^5} = 1 \times 10^9$$

Solving for $D_{eq} = 0.0505 \text{m}$

Assume $\varepsilon = 0.2 \text{mm}$ and hydraulically rough flow, $\varepsilon/D = 0.004$

initial $f = 0.028$ using 20.3 = $h_f = f \frac{L V^2}{D 2g}$

$V = 1.475 \text{m/s}$  $Re = 75997$

Recalculating $f = 0.03$  $V = 1.425 \text{ m/sec}$

Discharge $Q = 0.002855 \text{ m}^3 / \text{sec}$
\[ \frac{L_{eq}}{D_{eq}^5} = \sum_{i=1}^{n} \frac{L_i}{D_i^5} \]

choose either \( D_{eq} \) or \( L_{eq} \), then compute the other from the equation

<table>
<thead>
<tr>
<th>Pipe</th>
<th>( \varepsilon ) (mm)</th>
<th>D (cm)</th>
<th>( \varepsilon/D )</th>
<th>L (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.24</td>
<td>8</td>
<td>0.003</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>0.12</td>
<td>6</td>
<td>0.002</td>
<td>150</td>
</tr>
<tr>
<td>3</td>
<td>0.20</td>
<td>4</td>
<td>0.005</td>
<td>80</td>
</tr>
</tbody>
</table>

Let’s use \( D_{eq} = 0.06 \) m

\[
L / (0.06)^5 = (100/0.08^5) + (150/0.06^5) + (80/0.04^5) = 1 \times 10^9
\]

Solving for \( L_{eq} = 778 \) m

Assume \( \varepsilon = 0.2 \) mm and hydraulically rough flow, \( \varepsilon/D = 0.0033 \)

Initial \( f = 0.027 \) using 20.3 = \( h_f = \frac{fL}{D} \frac{V^2}{2g} \)

\[ V = 1.067 \text{ m/s} \quad \text{Re} = 65275 \]

Recalculating \( f = 0.03 \) \( V = 1.012 \text{ m/sec} \)

Discharge \( Q = 0.00286 \text{ m}^3 / \text{sec} \)
• If desired minor losses may be expressed in terms of equivalent lengths and added to the actual length of pipe as:

\[ h_m = K_m \frac{V^2}{2g} = f \frac{L_{eq}}{D} \frac{V^2}{2g} \quad \text{Hence} \quad L_{eq} = K_m \frac{D}{f} \]

Where

• \( K_m \) = local loss coefficient, and
• \( f \) = friction factor of the pipe
• Then the pipe length should be taken as:
• \( L = L_{ac} + L_{eq} \)
Pipes in Parallel
• In order to increase the capacity of a pipeline system, pipes might be connected in parallel. For pipes connected in parallel:

• The discharges are additive:
• \( Q_A = Q_B = \sum Q_i, \ i=1,2,\ldots,n \)

• The Total head at junctions must have single value. Therefore the head loss in each branch must be the same:

• \( h_{f1}=h_{f2}=h_{f3} = \cdots = h_{fn} \)
Example

- Assume that the same three pipes of previous example are now in parallel with the same total loss of 20.3 m. Compute the total rate \( Q(\text{m}^3/\text{hr}) \), neglecting the minor losses.

<table>
<thead>
<tr>
<th>pipe</th>
<th>L (m)</th>
<th>D (cm)</th>
<th>( \varepsilon ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>8</td>
<td>0.24</td>
</tr>
<tr>
<td>2</td>
<td>150</td>
<td>6</td>
<td>0.12</td>
</tr>
<tr>
<td>3</td>
<td>80</td>
<td>4</td>
<td>0.20</td>
</tr>
</tbody>
</table>
- **Solution-I:**

Energy equation b/w A and B:

\[ H_A = H_B + h_L = H_B + h_f + h_m \]

no matter which route is followed b/w A and B

\[ H_A - H_B = 20.3 = f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g} = f_3 \frac{L_3}{D_3} \frac{V_3^2}{2g} \]
• Substituting the known L’s and D’s,

\[ 20.3 = 1250 f_1 \frac{V_1^2}{2g} = 2500 f_2 \frac{V_2^2}{2g} = 2000 f_3 \frac{V_3^2}{2g} \]

Since \( V_i \)'s and \( f_i \)'s are not known, assume **hydraulically rough regime**

<table>
<thead>
<tr>
<th>Pipe</th>
<th>( \varepsilon/D )</th>
<th>( F_0 )</th>
<th>( V )</th>
<th>( R_e )</th>
<th>( f_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.003</td>
<td>0.0262</td>
<td>3.49</td>
<td>273726</td>
<td>0.268</td>
</tr>
<tr>
<td>2</td>
<td>0.002</td>
<td>0.0234</td>
<td>2.61</td>
<td>153529</td>
<td>0.247</td>
</tr>
<tr>
<td>3</td>
<td>0.005</td>
<td>0.0304</td>
<td>2.56</td>
<td>100392</td>
<td>0.315</td>
</tr>
</tbody>
</table>

\[ R_e = \frac{VD}{v} \]
### Tabular presentation

#### Table 1: Pipe Characteristics

<table>
<thead>
<tr>
<th>Pipe</th>
<th>$\varepsilon/D$</th>
<th>$f_1$</th>
<th>$V$</th>
<th>$Re$</th>
<th>$f_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.003</td>
<td>0.268</td>
<td>3.46</td>
<td>271373</td>
<td>0.268</td>
</tr>
<tr>
<td>2</td>
<td>0.002</td>
<td>0.247</td>
<td>2.55</td>
<td>150000</td>
<td>0.247</td>
</tr>
<tr>
<td>3</td>
<td>0.005</td>
<td>0.315</td>
<td>2.52</td>
<td>98823</td>
<td>0.315</td>
</tr>
</tbody>
</table>

F's have converged.

#### Table 2: Flow Rate

<table>
<thead>
<tr>
<th>Pipe</th>
<th>$V$(m/s)</th>
<th>$Q$(m$^3$/s)</th>
<th>$Q$(m$^3$/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.46</td>
<td>0.0174</td>
<td>62.6</td>
</tr>
<tr>
<td>2</td>
<td>2.55</td>
<td>0.0072</td>
<td>26.0</td>
</tr>
<tr>
<td>3</td>
<td>2.52</td>
<td>0.0032</td>
<td>11.4</td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

Q = 100 m$^3$/hr
Equivalent pipe concept for parallel pipes

- Consider two pipes, A and B, connected in parallel:

For the equivalent pipe with length, \( L_{eq} \), and diameter, \( D_{eq} \):

- For the equivalent pipe with length, \( L_{eq} \), and diameter, \( D_{eq} \):
• \( h_{f1-2} = h_{fA} = h_{fB} \) and \( Q_1 = Q_A + Q_B = Q_2 \)
• From the Darcy-Weissbach equation:

\[
Q = \sqrt{\frac{h_f g \pi^2 D^5}{8fL}}
\]

• Therefore:

\[
Q_A = \sqrt{\frac{h_{fA} g \pi^2 D_A^5}{8f_A L_A}} \quad \text{and} \quad Q_B = \sqrt{\frac{h_{fB} g \pi^2 D_B^5}{8f_B L_B}} \quad \text{and hence}
\]

\[
\sqrt{\frac{h_{fC} g \pi^2 D_C^5}{8f_C L_C}} = \sqrt{\frac{h_{fA} g \pi^2 D_A^5}{8f_A L_A}} + \sqrt{\frac{h_{fB} g \pi^2 D_B^5}{8f_B L_B}}
\]
• Simplifying:

\[ \sqrt{\frac{D_C^5}{f_C L_C}} = \sqrt{\frac{D_A^5}{f_A L_A}} + \sqrt{\frac{D_B^5}{f_B L_B}} \]

Furthermore if \( f_C = f_A = f_B \), then

\[ \sqrt{\frac{D_C^5}{L_C}} = \sqrt{\frac{D_A^5}{L_A}} + \sqrt{\frac{D_B^5}{L_B}} \]

Generalizing for \( n \) pipes connected in parallel:

\[ \sqrt{\frac{D_{eq}^5}{L_{eq}}} = \sum_{i=1}^{n} \sqrt{\frac{D_i^5}{L_i}} \]

(1)