Laminar Flow in Pipes

- Fluid is incompressible and Newtonian.
- Flow is steady, fully developed, parallel and symmetric with respect to pipe axis.
- Pipe is straight pipe and has a constant diameter.
Laminar Flow in Pipes

- Momentum Equation
\[ pA - \left( p + \frac{dp}{dx} \right) A - \gamma Adx \sin \theta - \tau 2\pi rdx = 0 \quad \text{(Constant velocity thus } a=0) \]

\[- \frac{dp}{dx} dx A - \gamma dx \frac{dz}{dx} A - \tau 2\pi r dx = 0\]

(Divide both sides by \( A = \pi r^2 \))

\[- \frac{d(p + \gamma z)}{dx} = 2\tau \]

\[- \frac{dh}{dx} = - \frac{d(p + \gamma z)}{\gamma dx} = + \frac{2\tau}{\gamma r} \quad \left( \text{since } h = \frac{p}{\gamma} + z \right) \]
Boundary Conditions

\[- \frac{dh}{dx} = - \frac{d(p + \gamma z)}{\gamma dx} = + \frac{2\tau}{\gamma r}\]

when \( r = 0 \), \( \tau = 0 \)

\( r = r_0 \), \( \tau = \tau_w \)

\[\tau = \tau_w (1 - \frac{r}{r_0})\]

\( \tau_w \).....wall shear stress

equations are equally applicable to both laminar and turbulent flow in pipes
Laminar Flow

\[ \tau = +\mu \frac{du}{dy} = -\mu \frac{du}{dr} \quad (1) \]

\[ \tau = - \frac{d(p + \gamma z) \; r}{dx} \frac{r}{2} \quad (2) \]

\[ \frac{du}{dr} = + \frac{d(p + \gamma z) \; r}{dx} \frac{r}{2\mu} \]
Boundary Conditions

\[ \frac{du}{dr} = \frac{d(p + \gamma z)}{dx} \frac{r}{2\mu} \]

\( u = u(r) \) may be solved by integration

\[ u(r) = \frac{d(p + \gamma z)}{dx} \frac{r^2}{4\mu} + C \]

\( r = r_0 \); \( u = 0 \)

\( r = 0 \), \( u = u_{\text{max}} \)

\[ u = -\frac{d(p + \gamma z)}{dx} \frac{r_0^2}{4\mu} \left[ 1 - \left( \frac{r}{r_0} \right)^2 \right] \]

Parabolic profile Poiseuille Flow
- **Velocity:** 
  \[ u = u_{\text{max}} \left[ 1 - \left( \frac{r}{r_o} \right)^2 \right] = - \frac{d(p + \gamma z)}{dx} \frac{r_o^2}{4\mu} \left[ 1 - \left( \frac{r}{r_o} \right)^2 \right] \]

- **Average velocity:** 
  \[ V = \frac{Q}{A} = \frac{\int udA}{A} = \frac{u_{\text{max}}}{2} = - \frac{d(p + \gamma z)}{dx} \frac{r_o^2}{8\mu} \]

- **Maximum velocity:** 
  \[ u_{\text{max}} = - \frac{d(p + \gamma z)}{dx} \frac{r_o^2}{4\mu} \]

- **Wall shear stress:** 
  \[ \tau_w = \frac{4\mu V}{r_o} \]

- **Shear stress:** 
  \[ \tau = -\mu \frac{du}{dr} = \tau_w \frac{r}{r_o} \]

- **Flow rate:** 
  \[ Q = V A = - \frac{\pi r_o^4}{8\mu} \frac{d(p + \gamma z)}{dx} \]

- **Head loss:** 
  \[ \frac{h_f}{L} = - \frac{dh}{dx} = - \frac{d(p + \gamma z)}{\gamma dx} \]
Turbulent Flow: \( Re \geq 4000 \)

Velocity very small near wall
Thus flow must be laminar!!
This region is called
Viscous sub-layer
(a) smooth wall and (b) a rough wall.
Shear velocity

\[ u_* = \sqrt{\frac{\tau_0}{\rho}} \]

Average velocity in laminar flow

\[ V = \frac{D u_*^2}{8\nu} \]

Viscous sublayer thickness

\[ \delta = 11.6 \frac{\nu}{u_*} \]

Smooth wall

\[ k_s < 11.6 \frac{\nu}{u_*} \]

Rough wall

\[ k_s > 70 \frac{\nu}{u_*} \]
Laminar and Turbulent

Laminar

$$u(r)$$

Turbulent

$$\overline{u(r)}$$

$$\tau_{w,turb} > \tau_{w,\text{lam}}$$
Comparison of laminar and turbulent flow

Laminar
- Can solve exactly. Flow is steady.
- Velocity profile is parabolic.
- Pipe roughness not important.

Turbulent
- *Cannot* solve exactly (too complex).
- Flow is unsteady, but it is steady in the mean.
- Mean velocity profile is fuller (shape more like a top-hat profile, with very sharp slope at the wall).
- $V_{\text{avg}}$ 85% of $U_{\text{max}}$ (depends on Re a bit).
- Pipe roughness is very important.
- No analytical solution, but there are some good semi-empirical expressions that approximate the velocity profile shape.
Darcy Weisbach Equation

- Consider a steady fully developed flow in a prismatic pipe ($A = \text{constant along centerline}$)
Relation between wall shear stress and head loss

\[ Q = V_1A_2 = V_2A_2 = VA = \text{Constant} \]

\[ p_1A_1 - p_2A_2 + W\sin\theta - F_f = 0 \]

\[ W\sin\theta = \gamma A L \sin\theta \]
\[ \gamma A L \sin\theta = \gamma A (z_1 - z_2) \]

\[ F_f = \tau_w PL \]

\[ V_2 = V_1 \]

\[ p_1A_1 - p_2A_2 + \gamma A (z_1 - z_2) - \tau_w PL = 0 \]

\[ A = A_2 = A_1 \]

\[ 1/(A\gamma) \]

\[ z_1 + \frac{p_1}{\gamma} - z_2 - \frac{p_2}{\gamma} = \frac{\tau_w LP}{\gamma A} \]
Relation between wall shear stress and head loss

\[
z_1 + \frac{p_1}{\gamma} - z_2 - \frac{p_2}{\gamma} = \frac{\tau_w L P}{\gamma A}
\]

\[A = \frac{\pi}{4} D^2 \quad P = \pi D \quad R_h = \frac{A}{P} = \frac{D}{4}\]

\[
\frac{\tau_w L P}{\gamma A} = \frac{\tau_w L}{\gamma R_H}
\]

\[
z_1 + \frac{p_1}{\gamma} - z_2 - \frac{p_2}{\gamma} = \frac{\tau_w L}{\gamma R_H}
\]

Hydraulic Radius
Relation between wall shear stress and head loss

\[ z_1 + \frac{p_1}{\gamma} - z_2 - \frac{p_2}{\gamma} = \frac{\tau_w L}{\gamma R_H} \]

Energy equation between section 1 and 2

\[ z_1 + \frac{p_1}{\gamma} - z_2 - \frac{p_2}{\gamma} = h_f \]

\[ h_f = \frac{\tau_w L}{\gamma R_H} = \frac{2\tau_w L}{\gamma R} = \frac{4\tau_w L}{\gamma D} \]

\[ h_f = \frac{4\tau_w L}{\gamma D} \]
Darcy – Weisbach Friction Factor

Laminar Flow: \( \text{Re} \leq 2000 \)

\[
h_f = \frac{4\tau_w L}{\gamma D}
\]

\[
\tau_w = \frac{4\mu V}{r_o}
\]

\[
h_f = \frac{8L\mu V_{av}}{\gamma r_o^2} = \frac{32L\mu V}{\gamma D^2} = \frac{2V}{2V} = \frac{32L\mu V}{\gamma D^2} = \frac{64 L V^2}{R_e D 2g}
\]

\[
f = \frac{64}{R_e}
\]
Friction factor for Turbulent Flows

For hydraulically smooth pipe,
\[ f = f(Re) \text{ only} \]

For frictionally transition zone:
\[ f = f(Re, \varepsilon/D) \]

For fully rough pipe:
\[ f = f(\varepsilon/D) \text{ only}. \]
Formula for friction factors in Turbulent flows

Smooth Pipe and Hydraulically Smooth Flow

\[
\frac{1}{\sqrt{f}} = -2 \log \left( \frac{2.51}{R_e \sqrt{f}} \right)
\]

\[ f = \text{func}(R_e) \]

Colebrook - White – Transitional Flow

\[
\frac{1}{\sqrt{f}} = -2 \log \left( \frac{2.51}{R_e \sqrt{f}} + \frac{\varepsilon}{3.7D} \right)
\]

\[ f = \text{func}(R_e, \frac{\varepsilon}{D}) \]

Rough Pipe-Hydraulically Rough Flow

\[
\frac{1}{\sqrt{f}} = -2 \log \left( \frac{\varepsilon}{3.7D} \right)
\]

\[ f = \text{func}(\frac{\varepsilon}{D}) \]
Swamee – Jain Formula (Explicit)

\[ f = \frac{1.325}{\left[ \ln \left( \frac{3.7 \varepsilon}{3.7D} + \frac{5.74}{R_e^{0.9}} \right) \right]^2} \]

for the range of \( 10^{-6} < \varepsilon/D < 10^{-2} \) and \( 5000 < R_e < 10^8 \)
Moody’s Diagram
Moody diagram. (From L.F. Moody, *Trans. ASME*, Vol.66,1944.) (Note: If $e/D = 0.01$ and $Re = 10^4$, the dot locates $f = 0.043$.)
**Determination of Friction Loss** $(h_f)$:

- **Darcy - Weisbach Equation**

  $$h_f = f \frac{L \ V^2}{D \ 2g} = f \frac{L \ 16 \ Q^2}{D^5 \ \pi^2 \ 2g} = KQ^2$$

- **Hazen-Williams Equation**

  $$h_f = \frac{6.8}{C^{1.85}} \frac{L \ V^{1.85}}{D^{1.165}} = \frac{10.6}{C^{1.85}} \frac{L \ Q^{1.85}}{D^{4.87}} = KQ^{1.85}$$
# Roughness Coefficients

<table>
<thead>
<tr>
<th>Material</th>
<th>Hazen-Williams C</th>
<th>Manning’s Coefficient n</th>
<th>Darcy-Weisbach Roughness Height ε (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asbestos cement</td>
<td>140</td>
<td>0.011</td>
<td>0.0015</td>
</tr>
<tr>
<td>Brass</td>
<td>135</td>
<td>0.011</td>
<td>0.0015</td>
</tr>
<tr>
<td>Brick</td>
<td>100</td>
<td>0.015</td>
<td>0.6</td>
</tr>
<tr>
<td>Cast-iron, new</td>
<td>130</td>
<td>0.012</td>
<td>0.26</td>
</tr>
<tr>
<td>Concrete:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steel forms</td>
<td>140</td>
<td>0.011</td>
<td>0.18</td>
</tr>
<tr>
<td>Wooden forms</td>
<td>120</td>
<td>0.015</td>
<td>0.6</td>
</tr>
<tr>
<td>Centrifugally spun</td>
<td>135</td>
<td>0.013</td>
<td>0.36</td>
</tr>
<tr>
<td>Copper</td>
<td>---</td>
<td>0.022</td>
<td>45</td>
</tr>
<tr>
<td>Corrugated metal</td>
<td>120</td>
<td>0.016</td>
<td>0.15</td>
</tr>
<tr>
<td>Galvanized iron</td>
<td>140</td>
<td>0.011</td>
<td>0.0015</td>
</tr>
<tr>
<td>Glass</td>
<td>135</td>
<td>0.011</td>
<td>0.0015</td>
</tr>
<tr>
<td>Lead</td>
<td>150</td>
<td>0.009</td>
<td>0.0015</td>
</tr>
<tr>
<td>Plastic</td>
<td>148</td>
<td>0.010</td>
<td>0.0048</td>
</tr>
<tr>
<td>Steel:</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Coal-tar enamel</td>
<td>145</td>
<td>0.011</td>
<td>0.045</td>
</tr>
<tr>
<td>New unlined</td>
<td>110</td>
<td>0.019</td>
<td>0.9</td>
</tr>
<tr>
<td>Riveted</td>
<td>120</td>
<td>0.012</td>
<td>0.18</td>
</tr>
<tr>
<td>Wood stave</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
h_f = \frac{6.8}{C^{1.85}} \left( \frac{L}{D^{1.165}} \right)^{1.85}
\]

\[
h_f = f \frac{L}{D} \frac{V^2}{2g}
\]
Total Head Loss

\[ h = h_f + h_m \]

- \( h_f \): Friction (Viscous, Major) loss
- \( h_m \): Local (Minor) loss
## COMPUTATION OF FLOW IN SINGLE PIPES

G- Given    D- to be determined

<table>
<thead>
<tr>
<th>Variable</th>
<th>Type of the problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>Type I</td>
</tr>
<tr>
<td>Fluid</td>
<td>G</td>
</tr>
<tr>
<td>Viscosity</td>
<td>G</td>
</tr>
<tr>
<td>Diameter</td>
<td>G</td>
</tr>
<tr>
<td>Length</td>
<td>G</td>
</tr>
<tr>
<td>Roughness</td>
<td>G</td>
</tr>
<tr>
<td>Flowrate, or</td>
<td>G</td>
</tr>
<tr>
<td>Average velocity</td>
<td></td>
</tr>
<tr>
<td>Pressure Drop,</td>
<td>D</td>
</tr>
<tr>
<td>or Head loss</td>
<td></td>
</tr>
</tbody>
</table>
1) **Determination of Head Loss (Type I)**

\[ H_1 = H_2 + h_\lambda \]  
and \[ h_\lambda = h_f \]  
since \( h_m = 0 \)

\[ h_f = H_1 - H_2 \]

\[ H = z + \frac{p}{\gamma} + \frac{V^2}{2g} \]

\[ h_f = f \left( \frac{L}{D} \right) \frac{V^2}{2g} \]  
since \( V = \frac{Q}{A} \) then \[ h_f = \frac{8fL}{g\pi^2D^5} Q^2 \]

- **Given:** \( Q \) (or \( V \)), \( L \), \( D \), \( v \), \( \varepsilon \)  
  - **Find:** \( h_f \)

1) \[ V = \frac{4Q}{\pi D^2} \]  
(or \( Q = \frac{V\pi D^2}{4} \))

2) \[ R_e = \frac{VD}{v} = \frac{4Q}{\pi D v} \]

3) \( \varepsilon/D \)

4) \( f(Re,\varepsilon/D) \) is determined  
(from Moody Chart or Eqs.)

5) \( h_f \) is computed
Example 1 (Type-I problem):

• A galvanized iron pipe with a roughness height of 5x10^{-6} m with a diameter of 0.05 m and a length of 100 m carries a discharge of 0.003 m3/s. Calculate the head loss.

• Cross section area  \( A = 3.14159 \times (0.025) \times (0.025) = 0.001963 \) m²

• Velocity  \( V = \frac{0.003}{0.001963} = 1.528 \) m/s

• Reynolds Number  \( R_e = \frac{VD}{\nu} = \frac{4Q}{\pi D \nu} = 1.528 \times 0.05 \times 10^6 = 76,394 \)

• Iron Pipe  \( \Rightarrow \) Relative roughness = 5x10^{-6} / 0.05 = 0.0001

• \( f = 0.0195 \) from the Chart
Moody diagram. (From L.F. Moody, *Trans. ASME*, Vol.66,1944.) (Note: If $e/D = 0.01$ and $Re = 10^4$, the dot locates $f = 0.043$.)
**Example 1**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>π</td>
<td>3.141592</td>
</tr>
<tr>
<td>g</td>
<td>9.81</td>
</tr>
<tr>
<td>ρ</td>
<td>1000</td>
</tr>
<tr>
<td>μ</td>
<td>0.001</td>
</tr>
<tr>
<td>ν</td>
<td>0.000001</td>
</tr>
</tbody>
</table>

A galvanized iron pipe with a roughness height of 0.000005 m with a diameter of 0.05 m and a length of 100 m carries a discharge of 0.003 m³/s.

its x-section is 0.001963 m² and water flows with a velocity of 1.528 m/s.

Flow Reynolds number is 76394 and the relative roughness is 0.0001.

using S-J formula the friction factor is obtained as 0.01941

The head loss in the pipe is found to be 4.620 m.
2) Determination of average velocity (Type II)

Given:

\[ h_f, L, D, \nu, \varepsilon \]

Find:

\[ V \]

Since \( f \) depends on \( V \) through \( Re \), and \( V \) is unknown a priori, iteration is needed.

\[ R_e = \frac{VD}{\nu} = \frac{4Q}{\pi D \nu} \]

Moody Chart or Eqs.
Solution procedure (Type II):

Given: $h_f$, $L$, $D$, $\nu$, $\varepsilon$  
Find : $V$

1. Calculate relative roughness
2. Select friction factor, 
   *(assume completely rough turbulent flow)*; $f(i) = f(0)$
3. Calculate velocity;
4. Calculate Reynolds number;
5. Determine $f$ by using data from Step1 and 4; $f(i+1)$ (use Moody Chart, or Equation)
6. Check if $f(i+1) = f(i)$?
7. no, go to step 3 with $f(i+1)$
8. yes, continue
9. Calculate $Q$ or $V$

$$f(i) = f(0)$$

$$\frac{\varepsilon}{D}$$

$$V = \sqrt{\frac{h_f D^2 g}{fL}}$$

$$R_e = \frac{VD}{\nu}$$

$$Q = V \frac{\pi D^2}{4}$$
## Iteration Table

- **Given:** \( hf, L, D, \nu, \varepsilon \).
- **Find:** \( V \)

### Formula

\[
V = \sqrt{\frac{hfD^2g}{fL}} \\
Re = \frac{VD}{\nu}
\]

<table>
<thead>
<tr>
<th>( f(i) )</th>
<th>( V )</th>
<th>( Re )</th>
<th>( f(i+1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(0) ) Assumed</td>
<td>calculated</td>
<td>calculated</td>
<td>( f(1) )-determined</td>
</tr>
<tr>
<td>( f(1) )</td>
<td>calculated</td>
<td>calculated</td>
<td>( f(2) )-determined</td>
</tr>
<tr>
<td>( f(2) )</td>
<td>calculated</td>
<td>calculated</td>
<td>( f(3) )-determined</td>
</tr>
</tbody>
</table>

- **iteration is stopped** when \( f(i) = f(i+1) \)

*obtained from Moody Chart, or determined using equations.*
Example 2 (Type-II problem):

- Example 2.2 (Type-II problem): A galvanized iron pipe with a roughness height of $5 \times 10^{-6}$ m with a diameter of 0.05 m and a length of 100 m has experienced head loss of 10 m. Calculate the flow rate.

### EXAMPLE

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>-2.718282</td>
</tr>
<tr>
<td>$\pi$</td>
<td>3.141592</td>
</tr>
<tr>
<td>$g$</td>
<td>9.81</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1000</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.001</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.000001</td>
</tr>
</tbody>
</table>

for water

- A galvanized iron pipe with a roughness height of $0.000005$ m with a diameter of $0.05$ m and a length of $100$ m has experienced head loss of $10$ m. Its x-section is $0.00196$ m$^2$ and the relative roughness is $0.0001$ carrying out an iterative solution procedure:

<table>
<thead>
<tr>
<th>Iteration</th>
<th>$f_i$</th>
<th>$V$</th>
<th>$Re$</th>
<th>$f_{i+1}$</th>
<th>$\Delta f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.02</td>
<td>2.215</td>
<td>110736.2</td>
<td>0.018105</td>
<td>-0.00189</td>
</tr>
<tr>
<td>1</td>
<td>0.01811</td>
<td>2.328</td>
<td>116387.2</td>
<td>0.017944</td>
<td>-0.00016</td>
</tr>
<tr>
<td>2</td>
<td>0.01794</td>
<td>2.338</td>
<td>116909.4</td>
<td>0.017929</td>
<td>-1.4E-05</td>
</tr>
<tr>
<td>3</td>
<td>0.01793</td>
<td>2.339</td>
<td>116956.2</td>
<td>0.017928</td>
<td>-1.3E-06</td>
</tr>
<tr>
<td>4</td>
<td>0.01793</td>
<td>2.339</td>
<td>116960.4</td>
<td>0.017928</td>
<td>-1.1E-07</td>
</tr>
<tr>
<td>5</td>
<td>0.01793</td>
<td>2.339</td>
<td>116960.8</td>
<td>0.017928</td>
<td>-1E-08</td>
</tr>
</tbody>
</table>

hence the discharge is obtained as $0.00459$ m$^3$/s
Example 2 (Type-II problem):

- Example 2.2 (Type-II problem): A galvanized iron pipe with a roughness height of $5 \times 10^{-6}$ m with a diameter of 0.05 m and a length of 100 m has experienced head loss of 10 m. Calculate the flow rate.

\[
D = 0.05 \text{ m} \\
g = 9.81 \text{ m/s}^2 \\
L = 100 \text{ m} \\
h_f = 10 \text{ m} \\
\varepsilon = 5 \times 10^{-6}
\]

\[
\begin{align*}
V &= \sqrt{\frac{h_f D^2 g}{fL}} \\
R_e &= \frac{VD}{\nu} \\
\frac{\varepsilon}{D} &= \frac{5 \times 10^{-6}}{0.05} = 0.0001 \\
f &= 0.012 \text{ assuming fully rough flow}
\end{align*}
\]

\[
V^2 = \left[ \frac{(10 \times 0.05 \times 2 \times 9.81)}{(0.012 \times 100)} \right] = 8.175 \quad V = 2.8592 \text{ m/s}
\]

\[
R_e = 2.8592 \times 0.05 / 10^{-6} = 1.43 \times 10^5
\]

\[
f = 0.0165 \quad \text{Since not the same with the previous continue iteration}
\]

<table>
<thead>
<tr>
<th>Initial f</th>
<th>V (m/s)</th>
<th>Re (10^6)</th>
<th>Final f</th>
<th>Del f</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.012</td>
<td>2.8592</td>
<td>0.143</td>
<td>0.0165</td>
<td>0.0045</td>
</tr>
<tr>
<td>0.0165</td>
<td>2.4383</td>
<td>0.122</td>
<td>0.017</td>
<td>0.0005</td>
</tr>
<tr>
<td>0.017</td>
<td>2.4022</td>
<td>0.120</td>
<td>0.017</td>
<td>0</td>
</tr>
</tbody>
</table>
Moody diagram. (From L.F. Moody, *Trans. ASME*, Vol.66,1944.) (Note: If $e/D = 0.01$ and $Re = 10^4$, the dot locates $f = 0.043$.)
3) Determination of Diameter (Type III)

- Given: \( h_f, L, Q, \nu, \varepsilon \). Find: \( D \)

1. Assume \( f(i) = f(0) \) (arbitrarily 0.02)

2. Calculate pipe diameter
   \[
   D = 5 \sqrt{f \frac{8LQ^2}{h_f \pi^2 g}} = \left( \frac{8LQ^2}{h_f \pi^2 g} \right)^{1/5} f^{1/5}
   \]

3. Calculate Reynolds number
   \[
   R_e = \frac{VD}{\nu} = \frac{4Q}{\pi D \nu}
   \]

4. Calculate relative roughness
   \[
   \frac{\varepsilon}{D}
   \]

5. Determine friction factor, \( f(i+1) \) use Moody Chart or Equations

6. Check if \( f(i+1) = f(i) \) ; ?

7. if no, go to step 2 with \( f(i+1) \)

8. if yes, stop. Diameter Calculated at Step 2 is the result.

9. Select the next larger commercially available pipe diameter size
\[ D = \sqrt[5]{\frac{8LQ^2}{h_\lambda \pi^2 g}} = \left( \frac{8LQ^2}{h_\lambda \pi^2 g} \right)^{1/5} f^{1/5} \]

\[ R_e = \frac{VD}{v} = \frac{4Q}{\pi D v} \]

\[ \frac{\varepsilon}{D} \]

<table>
<thead>
<tr>
<th>( f^{(i)} )</th>
<th>D</th>
<th>( R_e )</th>
<th>( \varepsilon/D )</th>
<th>( f^{(i+1)*} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f^{(0)} )</td>
<td>Calculated</td>
<td>Calculated</td>
<td>Calculated</td>
<td>( f^{(1)} )-determined</td>
</tr>
<tr>
<td>Assumed</td>
<td></td>
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</tr>
<tr>
<td>( f^{(1)} )</td>
<td>Calculated</td>
<td>Calculated</td>
<td>Calculated</td>
<td>( f^{(2)} )-determined</td>
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<tr>
<td>( f^{(2)} )</td>
<td>Calculated</td>
<td>Calculated</td>
<td>Calculated</td>
<td>( f^{(3)} )-determined</td>
</tr>
<tr>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( \ldots )</td>
<td>\textit{iteration is stopped}</td>
<td>\textit{stopped}</td>
<td>\textit{when}</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( f^{(i)} )</td>
<td>Calculated</td>
<td>Calculated</td>
<td>Calculated</td>
<td>( f^{(i+1)} )-determined</td>
</tr>
</tbody>
</table>
Example 2.3 (Type-III problem):
A galvanized iron pipe with a roughness height of 0.00005 m and a length of 100 m under the head loss of 10 m delivers discharge of 0.003 m$^3$/s. Calculate pipe diameter.

<table>
<thead>
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<th>EXAMPLE</th>
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<tr>
<td>$e$</td>
</tr>
<tr>
<td>$\pi$</td>
</tr>
<tr>
<td>$g$</td>
</tr>
<tr>
<td>$\rho$</td>
</tr>
<tr>
<td>$\mu$</td>
</tr>
<tr>
<td>$v$</td>
</tr>
</tbody>
</table>

A galvanized iron pipe with a roughness height of $\varepsilon$ m 0.00005 with a length of $L$ m 100 under the head loss of $h_f$ m 10 delivers discharge of $Q$ m$^3$/s 0.003

<table>
<thead>
<tr>
<th>carrying out an iterative solution procedure</th>
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</thead>
<tbody>
<tr>
<td>$i$</td>
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<td>0</td>
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<tr>
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</tr>
<tr>
<td>2</td>
</tr>
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<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
</tbody>
</table>

one obtains the friction factor and velocity (m/s)

<table>
<thead>
<tr>
<th>hence the discharge is obtained as</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
</tr>
</tbody>
</table>