MIDTERM EXAM
31 MARCH 2009
15:00
ANFI G
BRANCHING PIPES
Junction Problems

Res. 1

h₁

HGL

Res. 2

h₂

Res. 3

h₃

Junction, j

± 0
Junction Problems

Q_1 + Q_2 + Q_3 = 0

(flows towards the junction are considered to be positive!)

h_j = z_j + p_j / γ

h_{ℓ,i} = h_{f,i} + h_{m,i} = z_i - h_j
Identify the problem

• Given

✓ Reservoir elevations, $z_1, z_2, z_3$
✓ Pipe characteristics: $L_1, L_2, L_3, D_1, D_2, D_3, \varepsilon_1, \varepsilon_2, \varepsilon_3$

• Determine

✓ The flow rate $Q_1, Q_2, Q_3$
✓ and direction on each pipe
✓ Junction head: $h_j$ (piozometer head)
Solution Procedure

- Assume an appropriate direction for the flow in each pipe
- Assume a junction head
  - If friction factor is unknown
    - Make best assumption for the initial value of the friction factor (fully turbulent rough flow)
- Write energy equation from each reservoir to the junction
  - Calculate the flow rate for each pipe by using energy equation
- Put into continuity equation the determined discharge values
- Check that the continuity equation is satisfied or not
- If not change the junction head
Example

Reservoir B

Reservoir A

Reservoir C

85 m
D=200 cm
concrete

90 m

120 m
D=100 cm
concrete

20 m

120 m
D=100 cm
concrete

2

1

3

160 m
• Assume $Q_1$ and $Q_2$ “+” and $Q_3$ “-”
• Neglect all minor losses

Energy conservation b/w A&J: $h_A = h_J + h_{l,AJ} = h_J + h_{f,AJ}$
Energy conservation b/w B&J: $h_B = h_J + h_{l,BJ} = h_J + h_{f,BJ}$
Energy conservation b/w J&C: $h_J = h_C + h_{l,JC} = h_C + h_{f,JC}$

Assume $h_J$ and check if $Q_1 + Q_2 + Q_3 = 0$.
If not, iterate by assuming new $h_J$ until $\sum Q_i = 0$ checks.
Assume

\( h_J = 100 \text{m (elevation+50 m of pressure head)} \)

\[ h_{f,AJ} = \frac{8fLQ^2_1}{\pi^2 gD^5_1} = 9.92 \times f_1 \times Q^2_1 \]

\[ h_{f,BJ} = 0.22 \times f_2 \times Q^2_2 \]

\[ h_{f,CJ} = 9.92 \times f_3 \times Q^2_3 \]

hence b/w A and J \( 160 - 100 = 60 = 9.92 \times f_1 \times Q^2_1 \)

b/w B and J \( 140 - 100 = 40 = 0.22 \times f_2 \times Q^2_2 \)

b/w J and C \( 100 - 0 = 100 = 9.92 \times f_3 \times Q^2_3 \)

* Assume hydraulically rough flow, \( f = f(\varepsilon/D) \)
Concrete pipe  \( \varepsilon = 0.0001 \) \( Q_1 + Q_2 = Q_3 \)

- **\( h_J = 100\text{m} \)**

<table>
<thead>
<tr>
<th>( \varepsilon/D )</th>
<th>( f_i )</th>
<th>( Q_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>0.02</td>
<td>17.39</td>
</tr>
<tr>
<td>0.0005</td>
<td>0.018</td>
<td>100.50</td>
</tr>
<tr>
<td>0.001</td>
<td>0.02</td>
<td>22.57</td>
</tr>
</tbody>
</table>

17.39 + 100.50 >> 22.57
Therefore increase \( h_J \)

- **\( h_J = 130\text{m} \)**

<table>
<thead>
<tr>
<th>( f_i )</th>
<th>( Q_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>12.30</td>
</tr>
<tr>
<td>0.018</td>
<td>50.25</td>
</tr>
<tr>
<td>0.02</td>
<td>25.59</td>
</tr>
</tbody>
</table>

12.30 + 50.25 > 25.29
Therefore increase \( h_J \)

- **\( h_J = 139\text{m} \)**

<table>
<thead>
<tr>
<th>( f_i )</th>
<th>( Q_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>10.29</td>
</tr>
<tr>
<td>0.018</td>
<td>15.89</td>
</tr>
<tr>
<td>0.02</td>
<td>26.47</td>
</tr>
</tbody>
</table>

10.29 + 15.89 = 26.18 \( \approx 26.47 \)
NETWORK OF PIPES
**NETWORK OF PIPES**

** Conservation of Mass:**
Flow into each junction must be equal to the flow out of the junction.

**Energy Conservation:**
Algebraic sum of head losses around each and every loop must be zero.

**A, ..., I: JUNCTIONS/ NODES**
**EF: LINK/BRANCH**
Head Loss Equations

\[ h_\ell = h_f + h_m \]

\[ h_\ell = f \frac{L}{D^5} \frac{8Q^2}{\pi^2 g} + K_m \frac{8Q^2}{D^4 \pi^2 g} = K_{f_{\text{link}}} Q^2 + K_{m_{\text{link}}} Q^2 = K_{\text{link}} Q^2 \]

\[ h_l = K_{\text{link}} Q_{\text{link}} |Q_{\text{link}}| \]
Conservation of Energy around any loop

\[ h_{\text{loop}} = \sum_{i=1}^{N} K_i Q_i^n = \sum_{i=1}^{N} K_i (Q_o + \Delta Q)^n \]

\[ = \sum_{i=1}^{N} K Q_o^n + \sum_{i=1}^{N} nK\Delta QQ_o^{n-1} + \ldots \]

\[ h_{\text{loop}} = 0 \Rightarrow \sum_{i=1}^{N} K Q_o^n + \sum_{i=1}^{N} nK\Delta QQ_o^{n-1} = 0 \]

If \( n = 2 \) \( \Rightarrow n=2 \) for Darcy-Weisbach

\[ \Delta Q = -\frac{\sum_{i=1}^{N} KQ_o |Q_o|}{\sum_{i=1}^{N} 2K |Q_o|} \]
Solution Procedure

1) Assume flow distribution
   Condition: Flow distribution should satisfy conservation of mass at each node/junction
   Notation: $Q_o$

2) Calculate $\Delta Q$ for each loop

3) Check $\Delta Q$
   small magnitude
   YES: stop
   NO: Step 4

4) Correct $Q$ values
   $Q_{o,new} = Q_{o,old} + \Delta Q_{\text{loop}}$
Example

Given is the network shown in figure below. Find the discharges in each and every link.
Initial Guess

**Continuity** Flow into each junction must be equal flow out of the junction

### LOOP 1

<table>
<thead>
<tr>
<th>K Q</th>
<th>Q</th>
<th>2 K</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>AD</td>
<td>6<em>70</em>70=29400</td>
<td>2<em>6</em>70=840</td>
<td></td>
</tr>
<tr>
<td>AC</td>
<td>3<em>35</em>35 =3675</td>
<td>2<em>3</em>35=210</td>
<td></td>
</tr>
<tr>
<td>DC</td>
<td><em>-5</em>30*30 =-4500</td>
<td>2<em>5</em>30=300</td>
<td></td>
</tr>
<tr>
<td>Σ=28575</td>
<td></td>
<td>Σ=1350</td>
<td></td>
</tr>
</tbody>
</table>

\[ ΔQ_1 = -\frac{28575}{1350} = -21.17 \]

### LOOP 2

<table>
<thead>
<tr>
<th>K Q</th>
<th>Q</th>
<th>2 K</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>1<em>15</em>15 =225</td>
<td>2<em>1</em>15=300</td>
<td></td>
</tr>
<tr>
<td>BC</td>
<td>2<em>35</em>35=-2450</td>
<td>2<em>2</em>35=140</td>
<td></td>
</tr>
<tr>
<td>Σ=2799</td>
<td></td>
<td>Σ=253</td>
<td></td>
</tr>
</tbody>
</table>

**AC 35-21.17**
**Initial Guess**

**Flow Distribution After First Iteration**

**LOOP 1**

\[ \Delta Q_1 = -\frac{28575}{1350} = -21.17 \]

**AC 35-21.17**

**LOOP 2**

\[ \Delta Q_2 = \frac{2799}{253} = 11.06 \]
Second iteration

**LOOP 1**

<table>
<thead>
<tr>
<th>K Q</th>
<th>Q</th>
<th>2 K</th>
<th>Q</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AD</td>
<td>6<em>48.83</em>48.83</td>
<td>=14308</td>
<td>2<em>6</em>48.83</td>
<td>=586</td>
</tr>
<tr>
<td>AC</td>
<td>3<em>2.77</em>2.77</td>
<td>=23</td>
<td>2<em>3</em>2.77</td>
<td>=17</td>
</tr>
<tr>
<td>CD</td>
<td>5*-51.17*51.17</td>
<td>=-13090</td>
<td>2<em>5</em>51.17</td>
<td>=511</td>
</tr>
</tbody>
</table>

Σ=1241  Σ=1241  Σ=1241  Σ=1241  Σ=1114  Σ=1114  Σ=1114  Σ=1114

ΔQ₁ = \(-\frac{1241}{1114}\) = -1.114

AC 2.77-1.114

**LOOP 2**

<table>
<thead>
<tr>
<th>K Q</th>
<th>Q</th>
<th>2 K</th>
<th>Q</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>1<em>26.06</em>26.06</td>
<td>=679</td>
<td>2<em>1</em>26.06</td>
<td>=52</td>
</tr>
<tr>
<td>BC</td>
<td>2*-23.94*23.94</td>
<td>=-1146</td>
<td>2<em>2</em>23.94</td>
<td>=96</td>
</tr>
<tr>
<td>AC</td>
<td>3*-1.656*1.656</td>
<td>=-8</td>
<td>2<em>3</em>1.656</td>
<td>=10</td>
</tr>
</tbody>
</table>

Σ=-475  Σ=158

ΔQ₂ = \(\frac{475}{158}\) = 3.006
Final Distribution of Second Iteration

LOOP 1

\[ \Delta Q_1 = -\frac{1241}{1114} = -1.114 \]

AC 2.77 - 1.114 = 1.656

LOOP 2

\[ \Delta Q_2 = \frac{475}{158} = 3.006 \]

Attention: One more iteration is needed
HYDRAULICS AND OPERATION OF PUMPED DISCHARGE LINES

- **Pump**: adds energy to a fluid, resulting in an increase in pressure across the pump.

- **Turbine**: extracts energy from the fluid, resulting in a decrease in pressure across the turbine.
Turbine
Types of Pumps

(a) Radial flow pump

(b) Axial-flow pump
Centrifugal Pumps
The power gained $\Rightarrow P_f = \gamma Q H_p$

$H_p$  Head supply by pump (m)
$Q$  Discharge (m$^3$/s)
$P_f$  in Watts (Nm/s)

(in practice hp –horsepower- is used)

1hp = 745.7 Watts

$$P_f = \frac{\gamma Q H_p}{745.7}$$
A PUMP IS A MECHANICAL DEVICE WHICH ADDS ENERGY TO THE FLUID

The power required to derive the motor is \( bhp = \omega T \)

\[ P_f = \gamma Q H_p \]

bhp: brakehorsepower  
\( \omega \): shaft angular velocity  
T: shaft torque

There is a casing which provides passageway to the fluid.

- Axial
- Radial
- Mixed

\( \gamma \): fluid transfer coefficient
Static Suction Lift

Suction lift- Static head when the pump is located above the suction tank
Static Suction Head

Static head when the pump is located below the suction tank
Pump Head

- **Net Head**

\[
H = \left( \frac{P}{\rho g} + \frac{V^2}{2g} + z \right)_{\text{out}} - \left( \frac{P}{\rho g} + \frac{V^2}{2g} + z \right)_{\text{in}}
\]

\[
= EGL_{\text{out}} - EGL_{\text{in}}
\]

- **Water horsepower**

\[
\dot{W}_{\text{water horsepower}} = \dot{m}gH = \rho g \dot{V} H
\]

- **Brake horsepower**

\[
bhp = \dot{W}_{\text{shaft}} = \omega T_{\text{shaft}}
\]

- **Pump efficiency**

\[
\eta_{\text{pump}} = \frac{\rho g \dot{V} H}{\omega T_{\text{shaft}}}
\]
Efficiency

• If there were no losses: \( \text{bhp} = P_f \)

Yet, there are losses:

• mechanical losses in bearings and seals
• losses due to leakage of fluid
• other frictional and minor losses

\[
\eta = \frac{P_f}{\text{bhp}} = \frac{P_f}{P_p} = \frac{\gamma Q H_p}{P_p}
\]
Pump Power and efficiency

\[ P_p = \frac{\gamma Q H_p}{\eta} \]

where \( \eta \) is the efficiency.

So \( \eta \) must be maximized over a **wide** range of \( Q \).
Pump Performance

- **P**<sub>p</sub>, **η**, **H**<sub>p</sub>

- **Q**
- **H**
- **η**

- **shut off head**

- **rated capacity** (normal/design low rate)

<table>
<thead>
<tr>
<th>Q (lt/s)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (m)</td>
<td>70</td>
<td>67</td>
<td>62.5</td>
<td>57.5</td>
<td>51</td>
<td>43</td>
<td>32</td>
</tr>
<tr>
<td>η (%)</td>
<td>45</td>
<td>63</td>
<td>75</td>
<td>82</td>
<td>79</td>
<td>71</td>
<td></td>
</tr>
</tbody>
</table>
SELECTION OF A PUMP

Diagram showing a pump, valve, and water levels with a graph of pump performance and system curve.
SELECTION OF A PUMP

- Pump $H_p$ vs $Q$ must be known!
- System hydraulics must be known!
Consider a system with two reservoirs; water is supplied to the reservoir with higher elevation by means of a pumped line!

\[ H_1 = H_2 - H_p + h_f \]

\[ H_p = \Delta z + KQ^2 \]

\[ H_p = H_s + KQ^2 \]

\[
\begin{align*}
    z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} &= z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} - H_p + KQ^2 \\
\end{align*}
\]
Pumps in Parallel

- Head across each pipe (pump) is identical
- The total discharge through the pumping system is equal to the sum of individual discharges through each pump.
Head across each pipe (pump) is identical.
The total discharge through the pumping system is equal to the sum of individual discharges through each pump.
Pump in Series

System Demand

Pump A

Pump B
Construction of Pump Curve

For each discharge the head of the pump A and B are added each other.

Then a new curve is constructed

\[ H_{i(A+B)} = H_{i,A} + H_{i,B} \]

Discharge through each pump is identical!!!

\[ \eta_p = \frac{\gamma \left( \sum H_p \right) Q_D}{\sum P_p} \]
Pump in Series

\begin{equation}
Q_D = Q_A = Q
\end{equation}
EXAMPLE:

Determine the discharge and compute the power consumption.

**Pipeline Characteristics**

$L=2000\ m;\ D=0.2\ m;\ \varepsilon=0.0002m;\ \nu=1\times10^{-6}\ m^2/s$

when only one pump is operating

when both pumps are operating in series

when both pumps are operating in parallel

**Pump Characteristics**

<table>
<thead>
<tr>
<th>Q (lt/s)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (m)</td>
<td>70</td>
<td>67</td>
<td>62.5</td>
<td>57.5</td>
<td>51</td>
<td>43</td>
<td>32</td>
</tr>
<tr>
<td>$\eta\ (%)$</td>
<td>45</td>
<td>63</td>
<td>75</td>
<td>82</td>
<td>79</td>
<td>71</td>
<td></td>
</tr>
</tbody>
</table>
Single Pump

\[ R_e = \frac{VD}{v} = \frac{4Q}{\pi D v} \quad h_i = f \frac{L V^2}{D 2g} \]

| L (m) | 2000 |
|-------|
| D (m) | 0.2  |
| ε (m) | 0.0002 |
| ν (m²/s) | 0.000001 |

<table>
<thead>
<tr>
<th>Q (m³/s)</th>
<th>Re</th>
<th>f</th>
<th>hl</th>
<th>system</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.00</td>
<td>40.00</td>
</tr>
<tr>
<td>0.01</td>
<td>63662</td>
<td>0.0234</td>
<td>1.21</td>
<td>41.21</td>
</tr>
<tr>
<td>0.02</td>
<td>127324</td>
<td>0.0219</td>
<td>4.52</td>
<td>44.52</td>
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<tr>
<td>0.03</td>
<td>190986</td>
<td>0.0212</td>
<td>9.88</td>
<td>49.88</td>
</tr>
<tr>
<td>0.04</td>
<td>254648</td>
<td>0.0209</td>
<td>17.28</td>
<td>57.28</td>
</tr>
<tr>
<td>0.05</td>
<td>318310</td>
<td>0.0207</td>
<td>26.72</td>
<td>66.72</td>
</tr>
<tr>
<td>0.06</td>
<td>381972</td>
<td>0.0205</td>
<td>38.19</td>
<td>78.19</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>H</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>0</td>
</tr>
<tr>
<td>67</td>
<td>0.01</td>
</tr>
<tr>
<td>62.5</td>
<td>0.02</td>
</tr>
<tr>
<td>57.5</td>
<td>0.03</td>
</tr>
<tr>
<td>51</td>
<td>0.04</td>
</tr>
<tr>
<td>43</td>
<td>0.05</td>
</tr>
<tr>
<td>32</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Case 1: Single Pump

[Graph showing H vs Q for single pump with data points]

**Equations:**
- \[ \nu \pi = \nu \]
- \[ Q^4 V D = R_e g^2 \]
- \[ V D L f h_l = \]

**Diagram:**
- Lines representing system, Single Pump, and efficiency (eff.)
- X-axis: Q (m³/s)
- Y-axis: H (m)
SOLUTION
Q=35 lt/s
η=80%
(for Q=35 ℓ/s)
H=54m
P=23.2kW

\[ \eta_p = \frac{\gamma H_D \sum Q}{\sum P_P} \]
Case 2: Pump in Series

SOLUTION

Q=56 ℓ/s
η=75%
(for Q=56 ℓ/s)
∑H=72m
∑P=52.8 kW

<table>
<thead>
<tr>
<th>H(m)</th>
<th>Q(m^3/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>140</td>
<td>0</td>
</tr>
<tr>
<td>134</td>
<td>0.01</td>
</tr>
<tr>
<td>125</td>
<td>0.02</td>
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<tr>
<td>115</td>
<td>0.03</td>
</tr>
<tr>
<td>102</td>
<td>0.04</td>
</tr>
<tr>
<td>86</td>
<td>0.05</td>
</tr>
<tr>
<td>64</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Single Pump

H(m) | Q(m^3/s) |
------|----------|
70    | 0        |
67    | 0.01     |
62.5  | 0.02     |
57.5  | 0.03     |
51    | 0.04     |
43    | 0.05     |
32    | 0.06     |
Case 3: Pump in Parallel

SOLUTION

Q = 46 ℓ/s
η = 68%
(for Q = 23 ℓ/s)
∑H = 61 m
∑P = 40.5 kW
## Summary

<table>
<thead>
<tr>
<th></th>
<th>Single</th>
<th>Series</th>
<th>Parallel</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Q (lt/s)</strong></td>
<td>35</td>
<td>56</td>
<td>46</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(2*23)</td>
</tr>
<tr>
<td><strong>η (%)</strong></td>
<td>80</td>
<td>75</td>
<td>68</td>
</tr>
<tr>
<td><strong>H (m)</strong></td>
<td>54</td>
<td>72</td>
<td>61</td>
</tr>
<tr>
<td><strong>P (kW)</strong></td>
<td>23.2</td>
<td>52.7</td>
<td>40.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(2*20.24)</td>
</tr>
<tr>
<td><strong>P/Q (kW.s/lt)</strong></td>
<td>0.66</td>
<td>0.94</td>
<td>0.88</td>
</tr>
</tbody>
</table>
The pipelines through which flow is maintained by the action of gravity are known as **gravity pipelines**.

A control valve might be used to control the discharge and pressure.

\[ 0.5 \text{ m/s} < V < 2 \text{ m/s} \]

\[ (p/\gamma) > 5 \text{ m} \]
Because of limits set forth on velocity and pressure head there are lower and upper bounds for the discharge through a gravity pipeline. Consider the following schematic representation of a gravity pipeline system shown below:

Energy Equation between points (A) & (B) is:

\[ H_A = H_B + h_L, \text{ or in open form:} \]

\[
\begin{align*}
\frac{z_A}{\gamma} + h_{\text{res}} + \frac{P_A}{\gamma} + \frac{V_A^2}{2g} &= \frac{z_B}{\gamma} + h_B + \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + h_L 
\end{align*}
\]
\[ z_A + h_{\text{res}} + \frac{P_A}{\gamma} + \frac{V_A^2}{2g} = z_B + h_B + \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + h_L \]

\( P_A = P_B = 0, \ V_A = 0, \) and \( z_B = 0. \)

The maximum value of \( V = 2 \text{ m/s}. \)

\[
\frac{V_B^2}{2g} \frac{2^2}{2 \times 9.81} = 0.2 \text{ m} \quad \text{can be neglected}
\]

\[ h_f = 8f \frac{L}{\pi^2 g D^5} \quad Q^2 = KQ^2, \]

where \( K = 8f \frac{L}{\pi^2 g D^5} \) \( \quad \square \) \( \hat{\varepsilon} h_f = KQ^2 \)
Therefore, Energy Equation becomes:

\[ h_{\text{res}} + z_A = h_f = KQ_{\text{out}}^2 \Rightarrow Q_{\text{out}} = \left(\frac{h_{\text{res}} + z_A}{K}\right)^{1/2} \]

(note that since \( V < 2 \text{ m/s} \) velocity heads are neglected!)

A relationship between \( Q \) and the reservoir level can be obtained as:

\[ Q = \sqrt{h_{\text{res}} + z_A} / K \]
For a full reservoir \( h_{\text{res}} = h_{\text{max}} \rightarrow Q = Q_{\text{max}} \)

\[ h_A = h_{\text{max}} \quad Q = Q_{\text{max}} \quad (\text{since } z \text{ and } K \text{ are constants}) \]

\[ Q_{\text{max}} = \sqrt{\frac{h_{\text{max}} + Z_A}{K}} \]

maximum discharge that may occur = in the pipeline system is called

\[ \text{The system capacity} \]

For empty reservoir \( h_{\text{res}} = 0 \rightarrow Q = Q_{\text{min}} = Q_0 \)

\( Q_0 \) is the minimum flow rate, which will pressurize the pipe, in other words, it is the minimum flow rate for a full pipe flow.

Therefore \[ Q_0 = \sqrt{\frac{Z_A}{K}} \]
• If $Q_{\text{max}} < Q_{\text{in}}$, $h = h_{\text{max}}$; $Q_{\text{spill}} = Q_{\text{in}} - Q_{\text{max}}$

• If $Q_{\text{min}} < Q_{\text{in}} < Q_{\text{max}}$, $0 < h_{\text{res}} < h_{\text{max}}$; $Q_{\text{spill}} = 0$

• If $Q_{\text{in}} < Q_{\text{min}}$ to prevent free flow the valve is partially closed
The free surface flow may be prevented by use of a valve at the pipe exit.

Valves control the flow rate by providing a mean to adjust the overall system loss coefficient to the desired value.
• Consider the same system with a valve at the end of the pipe:

\[ H_A = H_B + h_f + h_V \], or in open form:
\[ z_A + h_{\text{res}} + \frac{P_A}{\gamma} + \frac{V_A^2}{2g} = z_B + h_B + \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + h_f + h_V \]

\( P_A = P_B = 0, \ V_A = V_B = 0, \) and \( z_B = 0. \)

Also \( h_f = KQ^2 \) & the local loss can also be written as \( h_V = KVQ^2 \)

Therefore:

\[ z_A + h_A = (K + KV)Q^2, \] solving for \( Q: \)

\[ Q = \sqrt{\frac{z_A + h_{\text{res}}}{K + KV}} \]
Valve Conditions

\[ Q = \sqrt{\frac{z_A + h_{\text{res}}}{K + K_V}} \]

- when valve is closed \( K_V \rightarrow \infty \) Therefore \( Q=0 \).
- Opening of the valve reduces the value of \( K_V \), producing a desired flow rate.
• The head loss in valves is mainly a result of dissipation of kinetic energy of a high speed portion of flow.

• when $Q_{in} < Q_{min}$, the control valve at the pipe exit will pressurize the flow in the pipe by dissipating the excess kinetic energy, $h_V$, and in turn increasing the water level in the reservoir.

• The magnitude of the valve loss depends on how much one needs to presurize the flow, that is the water depth in the reservoir.

• If the water level is to be kept constant to obtain a certain discharge, a control valve can be chosen accordingly.
Example

Consider the reservoir-pipe system given below, with following values. Reservoir depth \( h_{\text{max}} = 5 \text{m} \), \( z_A = 5 \text{m} \), \( L = 2000 \text{m} \), \( D = 0.8 \text{m} \), \( f = 0.02 \).

Determine:

- The system capacity, \( Q_{\text{max}} \).
- Minimum flowrate, \( Q_0 \).
- Spill flowrate, if \( Q_{\text{in}} = 1.2 \text{m}^3/\text{s} \).
- Valve loss, \( h_v \), if \( Q_{\text{in}} = 0.5 \text{m}^3/\text{s} \).
Solution

If minor losses except $h_v$ (valve loss) are neglected,

$$Q_{\text{out}} = \left( \frac{h_{\text{res}} + z_A}{K} \right)^{1/2}$$

$$K = \frac{8fL}{g\pi^2D^5} = 10.09$$

$$Q_{\text{max}} = \left( \frac{5 + 5}{10.09} \right)^{1/2} \approx 1.0 \text{ m}^3/\text{s}$$

$$Q_{\text{min}} = \left( \frac{5}{10.09} \right)^{1/2} \approx 0.70 \text{ m}^3/\text{s}$$

If inflow is 1.2 m$^3$/s $\rightarrow Q_{\text{spill}} = 0.20 \text{ m}^3/\text{s}$

$$h_{\text{res}} + z_A = h_v + KQ_{\text{in}}^2 \Rightarrow h_v \geq z_A - KQ_{\text{in}}^2$$

$$h_v \geq 5 - 10.09(0.5)^2$$

$$h_v \geq 2.48 \text{ m}$$
Selection of Pipe Diameter: Cost

- In design of gravity pipelines, an optimum diameter is selected that minimizes the capital cost (The estimated total cost) and operation cost.

Cost = Capital + Operation cost
Selection of Pipe Diameter: Velocity

- Depending on the type of pipe material a lower and an upper limits are set for the velocity as:
  
  \[ 0.5 \text{ m/s} < V < 2 \text{ m/s}; \]
Selection of Pipe Diameter: Pressure

- To prevent air entrainment minimum pressure head, \((p/\gamma)_{\text{min}}\) permitted along the pipeline is 5m.

- The Range of Pressure
  - Transmission line: \(20 - 30 \leq P/\gamma \leq 80 \) (m)
  - City Network: \(3 - 5 \leq P/\gamma \leq 80 \) (m)
Computation of Pipe Diameter For Gravity Pipelines
• Determine the control points (C,E,F) and their topographic elevations \( z_c, z_E, z_F \)
• Add the minimum required pressure head to these elevations (\( z_c \) and \( z_e \)).
• Determine the minimum energy grade line slope, between the reservoir and control points
Minimum Energy Slope

\[ S_1 < S_2 < S_3 \]

\[ S_1 = \frac{H_A - (z_c + p_{min} / \gamma)}{L_{AC}} \]
Pipe Diameter for A-C

- Compute the pipe diameter for line A-C using Darcy-Weisbach equation and select the nearest commercially available diameter

\[
D_{\text{computed}} = \left( \frac{8fQ^2}{g\pi^2 S_1} \right)
\]
Velocity Check

• Compute the Velocity

\[
V = \frac{Q}{\pi D^2 / 4}
\]

• If \( V_{\text{min}} < V < V_{\text{max}} \) then the selected diameter is used in the project

• If \( V < V_{\text{min}} \) a booster pump must be installed at the reservoir site to increase the velocity to \( V_{\text{min}} \)
Booster Pump Head

- If booster pump is installed, the head supplied by booster pump will be computed as

\[ H_A + H_P = z_c + \frac{p_{\text{min}}}{\gamma} + f \frac{L}{D} \frac{V_{\text{min}}^2}{2g} \]
Maximum Velocity Check

• If $V > V_{\text{max}}$ then the pipe diameter is increased to reduce the velocity

\[ \frac{\pi D^2}{4} \times V_{\text{max}} = Q \]
Pressure Check

• If $p/\gamma < p_{\text{max}}$ then the selected pipe diameter is used

• If $p/\gamma > p_{\text{max}}$ A pressure reduction pump must be installed
Determination of diameter (C-D-E)

- Compute the piozemeter head at C
- Calculate the diameter for C-D-E-F segment
- Note that E is the control point
- Compute the diameter of C-D-E
- Compute the diameter of the segment E-F
Control valve

• Install a control valve at point F and determine the necessary headloss at the valve to maintain the pressurized flow at segment E-F.
Example 2.12: Determine the flow capacity of the given gravity pipeline system. Use an air valve that operates under a pressure head of $p/\gamma = 5$ m.

\[ H_A = H_C + h_{AC} \quad ; \quad h_{AB} = h_{BC} = h_{L5000} = 8.263Q^2 \]

\[ 10 = 2 \times 8.263Q^2 \quad \Rightarrow \quad Q = 0.78 \text{ m}^3 / \text{s} \]

\[ H_C = H_D + h_V + h_{CD} \]

\[ 120 = 100 + h_V + 8.263 \times Q^2 \]

\[ h_V = 15 \text{ m} \]

(i.e at D at least 15 m of head must be dissipated.)

Solution:

\[ S_{AC} = \frac{130 - 120}{10000} = 0.001 \quad \quad S_{AD} = \frac{130 - 100}{15000} = 0.002 \]

Since $S_{AC} < S_{AD}$; $S_{AC}$ = 0.001 is the (milder) critical slope. Therefore at C an air valve is installed.