İstanbul Kültür University
Department of Computer Engineering

MAT 002 - Numerical Methods
Fall 2011-2012

First Midterm

March 22, 2012

Number: Name:

Directions – You have 110 minutes to complete the exam. Please do not leave the examination room in the first 30 minutes of the exam. There are five questions, of varying credit (100 points total). Indicate clearly your final answer to each question. You are allowed to use a calculator. During the exam, please turn off your cell phone(s). You cannot use the book or your notes. You have one page for "cheat-sheet" notes at the end of the exam papers. Do use the **radian mode** on your calculator when using the trigonometry buttons. Please use **five-decimal digit** in your calculations. The answer key to this exam will be posted on Department of Mathematics and Computer Science board after the exam.

Good luck! *Emel Yavuz Duman, PhD.*

Question 1.		Question 4.	
Question 2.		Question 5.	
Question 3.		MARK	

Question 1. $5 + 5 + 10$ *points*

(a) Show that the equation $x = 3 + \frac{\sin x}{2}$ has a solution in the interval $[3, 4]$.
Answer.

(b) Determine the number of iterations necessary to solve the equation $x = 3 + \frac{\sin x}{2}$ with accuracy 10^{-1} using $a_1 = 3$ and $b_1 = 4$.
Answer.

(c) Use the Bisection method to determine an approximation to the solution that is accurate to at least within 10^{-1} for the equation $x = 3 + \frac{\sin x}{2}$ where $3 \leqq x \leqq 4$.
Answer.

n	a_n	b_n	p_n	$f(p_n)$
1				
2				
3				
4				
5				

Question 2. $5 + 15$ *points*

(a) Find an interval $[a, b]$ containing a solution of $f(x) = \sin x - \frac{1}{10} \ln x$.

Answer.

(b) Use four steps of the Secant method to find a solution of $f(x) = \sin x - \frac{1}{10} \ln x$ in this interval determined in (a), by taking $p_0 = a$ and $p_1 = b$.

Answer.

n	p_n	$f(p_n)$
0		
1		
2		
3		

Question 3. 15 *points*

Use the Newton's method to approximate the root of function f given by

$$f(x) = \tan x + \ln x$$

by taking $p_0 = 0.5$, within $\varepsilon = 10^{-4}$.
Hint. $\frac{d}{dx} \tan x = \frac{1}{\cos^2 x}$.
Answer.

n	p_n	$f'(p_n)$	$f(p_n)$
0			
1			
2			
3			
4			

Question 4. $15 + (2 + 8)$ *points*

Let $f(x) = e^x \cos x$ and $x_0 = 0$.

(a) Find the third Taylor polynomial $P_3(x)$ and remainder $R_3(x)$ for f about $x_0 = 0$.

Answer.

(b) Use $P_3(0.05)$ to approximate the value of $f(0.05)$ and find an upper bound for this approximation by using $R_3(0.05)$.

Answer.

Question 5. $10 + 10$ *points*

(a) Show that $g(x) = \ln(2x + 1)$ has a fixed point in the interval $[1, 2]$. (Do not approximate the fixed point. Verify all conditions)

Answer.

(b) How many fixed point iteration would be required to locate this fixed point in $[1, 2]$ to accuracy of $\varepsilon = 10^{-11}$ with initial point $p_0 = 1$.

Answer.