EXAMPLE 6 : HYDRAULIC JUMP

A rectangular horizontal channel 2m. wide, carries a flow of 4 m$^3$/s. The depth water on the downstream side of the hydraulic jump is 1m.

a) What is the depth upstream?

b) What is the loss of head?

\[ F_1 = F_2 \rightarrow y_1 A_1 + \frac{Q^2}{g A_1} = y_2 A_2 + \frac{Q^2}{g A_2} \]

\[ y_1 (2 y_1) + \frac{4^2}{2 g y_1} = 1 \times (2 \times 1) + \frac{4^2}{2 g (2 	imes 1)} \]

\[ 2 y_1^2 + \frac{0.815}{y_1} = 2.815 \]

\[ y_1 = 0.311 \text{ m} \]

\[ V_1 = 4/(2 \times 0.311) = 6.43 \text{ m/s} \]

\[ V_2 = 4/(2 \times 1) = 2.0 \text{ m/s} \]

\[ y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + h_L \]

\[ h_{\text{loss}} = 2.42 - 1.20 = 1.22 \text{ m} \]
Example on Critical Flow : 1

A flow of 28 m$^3$/sec occurs in an earth-lined trapezoidal channel having base width 3.0m, side slopes 1V:2H and $n = 0.022$. Calculate the critical depth and critical slope.

At critical flow Froude number is equal to unity

$$F_r^2 = \frac{Q^2 T}{gA^3} = 1$$

$Q = 28$ m$^3$/s  $B= 3$  $n=0.022$  slope 1V:2H

$$A = 3y + 2y^2 ; T = 3+4y ; P = 3+2 \ y(5)^{1/2}$$

$$R = \frac{A}{P} = \frac{(3y + 2y^2)}{(3+4y)}$$

solving by trial and error $y_c = 1.495$m

From Manning’s

$$Q = \frac{A}{n} R^{2/3} S^{1/2}$$

Solving for $S_c = 0.0052$
A channel of rectangular section 3.5m wide, with n= 0.014 and $S_0= 0.001$ leads from a lake whose surface level is 6m above the channel bed at the lake outlet. Find the discharge in the channel.

The $Q_{\text{max}}$ would result from critical flow at the outlet!!!

At reservoir V small thus

$$E = y + \frac{V^2}{2g} = 6m$$

No head loss between sections.

For a rectangular channel at critical flow

$$E_c = y_c + \frac{y_c}{2} \quad \text{or} \quad y_c = 4 \text{ m}$$

Velocity head 2m or velocity $V_c = 6.264 \text{ m/s}$

$Q_{\text{max}} = 87.69 \text{ m}^3/\text{sec}$
Example on Critical Flow : 3

During a major flood events water flows over the top of a roadway. Determine the head on the broad-crested weir for a discharge of 300 m$^3$/s if the overflow section of the roadway is horizontal and 150m. long.

For a rectangular channel at critical flow

$$1 = \frac{Q^2 T_c}{gA_c^3} \quad \text{or} \quad y_c = \left( \frac{q^2}{g} \right)^{1/3}$$

$q = 300/150 = 2 \text{ m}^3/\text{sec per meter}$

$$y_c = \left( \frac{2^2}{g} \right)^{1/3} = 0.74 \text{ m}$$
Flow in Open Channels

DESIGN OF OPEN CHANNELS FOR UNIFORM FLOW
**Channel design for Uniform Flow**

- The basic problem is the economical proportioning of the cross section.
- Channel with given \( n \) and \( S_0 \) for a known \( Q \) the objective is to minimize the area.

\[
Q = \frac{A}{n} R^{2/3} S_0^{1/2} \quad \frac{Qn}{\sqrt{S_o}} = AR^{2/3} = \frac{A^{5/3}}{P^{2/3}} = K_o
\]

- If \( A \) is minimum \( \Rightarrow \) \( V \) maximum from continuity
  \( R \) maximum from Manning’s
  \( P \) minimum (\( R=A/P \))
Hydraulic Efficiency of Cross-sections

Conveyance of the channel section

\[
\frac{Qn}{\sqrt{S_o}} = AR^{2/3} = \frac{A^{5/3}}{P^{2/3}} = K_o
\]

It can be shown that the ideal section would be

Semicircle
Best Hydraulic Section

\[ \frac{Qn}{\sqrt{S_o}} = AR^{2/3} = \frac{A^{5/3}}{P^{2/3}} = K_o \]

A channel section having the least wetted perimeter for a given area has the maximum conveyance; such a section is known as the best hydraulic section.
Channel design for Uniform Flow

However, other economical concerns

- Total volume of excavation
- Cost of lining
- Construction techniques
- Scour in erodible bed
- Sedimentation for low V
- Short channels and variable $S_0$

may require changes
The best hydraulic section for a rectangular channel

Area $A = b \times y$
Perimeter $P = b + 2y$

Perimeter must be minimum for given area
$P = A/y + 2y$
$dp/dy = -A/y^2 + 2 = 0$
$A = 2 \times y^2$
$b = 2y^2/y = \boxed{b= 2y}$
Example 7

What are the most efficient dimensions (the best hydraulic section) for a concrete \( (n=0.012) \) rectangular channel to carry \( 3.5 \text{ m}^3/\text{s} \) at \( S_o=0.0006 \)?

Given: \( n=0.012 \) \( Q=3.5 \text{ m}^3/\text{s} \) \( S_o=0.0006 \)
Find \( b \) and \( y \).

\[
Q = \frac{A}{n} R^{2/3} S_o^{1/2} \quad Q = \frac{A}{n} \left( \frac{A}{P} \right)^{2/3} S_o^{1/2} \quad Q = \frac{b y}{n} \left( \frac{b y}{b + 2y} \right)^{2/3} S_o^{1/2}
\]

Best section \( b=2y \)

\[
Q = \frac{2y^* y}{n} \left( \frac{2y^* y}{2y + 2y} \right)^{2/3} S_o^{1/2} \quad Q = \frac{2y^2}{n} \left( \frac{y}{2} \right)^{2/3} S_o^{1/2}
\]

\[
Q = \frac{2}{n} \left( \frac{y}{y^3} \right)^{1/3} S_o^{1/2} \quad \left( \frac{y}{y^3} \right)^{8/3} = 1.36 \quad y = 1.123 \text{ m and } b = 2.246 \text{ m}
\]
Best Hydraulic Sections

<table>
<thead>
<tr>
<th>Section</th>
<th>Most efficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trapezoidal</td>
<td>Base &lt; depth</td>
</tr>
<tr>
<td>Rectangular</td>
<td>Width = 2 x depth</td>
</tr>
<tr>
<td>Triangular</td>
<td>No specific relationship</td>
</tr>
<tr>
<td>Circular</td>
<td>Semicircle if open</td>
</tr>
<tr>
<td></td>
<td>Circle if closed</td>
</tr>
</tbody>
</table>
Precautions

- Steep slopes cause high velocities which may create erosion in erodible (unlined) channels.

- Very mild slopes may result in low velocities which will cause silting in channels. (Sedimentation)

- The proper channel cross-section must have adequate hydraulic capacity for a minimum cost of construction and maintenance.
Typical Cross Sections

- The cross-sections of unlined channels are recommended as trapezoidal in shape with side slopes depending mainly on the kind of foundation material.

(considering construction techniques and equipment, and stability of side inclination, the United States Bureau of Reclamation (USBR) and the Turkish State Hydraulic Works (DSİ) suggest standard **1.5H:1V** side slopes for trapezoidal channels.)
# Recommended side slopes

<table>
<thead>
<tr>
<th>Material</th>
<th>Side Slope (H:V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock</td>
<td>Nearly vertical</td>
</tr>
<tr>
<td>Muck and peat soils</td>
<td>¼: 1</td>
</tr>
<tr>
<td>Stiff clay or earth with concrete lining</td>
<td>½:1 to 1:1</td>
</tr>
<tr>
<td>Earth with stone lining or earth for large channels</td>
<td>1:1</td>
</tr>
<tr>
<td>Firm clay or earth for small ditches</td>
<td>1.5:1</td>
</tr>
<tr>
<td>Loose sandy earth</td>
<td>2:1</td>
</tr>
<tr>
<td>Sandy loam or porous clay</td>
<td>3:1</td>
</tr>
</tbody>
</table>
Recommended side slopes
Kızılıkaya Dam

Typical tertiary (lined) and interceptor (unlined) canals

Cross-section for $Q > 5 \text{ m}^3/\text{s}$ in cut.
Recommended channel cross-sections (Kızılkaya, 1988).
DESIGN OF NONERODIBLE CHANNELS

- For nonerodible channels the designer simply computes the dimensions of the channel by a uniform-flow formula and then final dimensions on the basis of hydraulic efficiency, practicability, and economy.

**Minimum Permissible velocity**

- In the design of lined channels the minimum permissible velocity is considered
- to avoid deposition if water carries silt or debris
  \[ V_{\text{min}} = 0.75 \text{ m/s} \quad \text{(non-silting velocity)} \]
The determination of section dimensions for nonerodible channels, includes the following steps:

- All necessary information, i.e. the design discharge, the Manning’s n and the bed slope are determined.
- Compute the section factor, $Z$, from the Manning equation
- If the expressions for $A$ and $R$ for the selected shape are substituted in the equation, one obtains 3 unknowns ($b$, $y$, $z$) for trapezoidal sections, and 2 unknowns ($b$, $y$) for rectangular sections.

\[
Q = \frac{1}{n} AR^{2/3} S_o^{1/2}
\]

\[
A = (b+zy)y \quad P = b + 2y\sqrt{1+z^2}
\]

\[
Z = AR^{2/3} = \left[ \frac{(b + zy)y}{b + 2y\sqrt{1 + z^2}} \right]^{5/3}
\]

Various combinations of $b$, $y$ and $z$ can be found to satisfy the above section factor $Z$.

The final dimensions are decided on the basis of hydraulic efficiency, practicability and economy.
Methods and Procedures

1. Assume side slope $z$
2. get the value of $b$ from the experience curve,
3. Solve for $y$.

\[
Z = AR^{2/3} = \frac{[(b + zy)y]^{5/3}}{[b + 2y\sqrt{1 + z^2}]^{2/3}}
\]

Experience Curves showing bottom width and water depth of lined channels
Best Hydraulic Section:

- 1) substitute A and R for best hydraulic section in Eq(*),
- 2) Solve for y

Ex. Trapezoidal sections:

\[
\begin{align*}
    z & = \frac{1}{\sqrt{3}} \\
    A & = \sqrt{3} y^2 \\
    b & = \frac{2}{3} \sqrt{3} y \\
    R & = \frac{y}{2} \\
    T & = \frac{4}{3} \sqrt{3} y
\end{align*}
\]
Checks

1) In the proximity of critical depth, flow becomes unstable with excessive wave action, hence it is recommended that:

   for subcritical flows: \( y > 1.1y_c \) (or \( Fr < 0.86 \))

   for supercritical flows: \( y < 0.9y_c \) (or \( Fr > 1.13 \))

2) Check the minimum permissible velocity if the water carries silt. \( (V_{min} > 0.75 \text{ m/s}) \)
The freeboard, \( f \), is determined by an empirical equation
\[
f = 0.2 \ (1+y)
\]
where, \( f \) is the freeboard (m) \( y \) is the water depth (m)
or by the curves given in Figure 1 for irrigation canals for the USBR and DSI practices.
Finalization

1) Modify the dimensions for practicability

2) Add a proper freeboard to the depth of the channel section. Recommended freeboard for canals is given in figure.

3) Draw channel cross section and show dimensions and given parameter.
Open Channel Design Example 1a

- A trapezoidal channel carrying 11.5 m$^3$/s clear water is built with concrete (nonerodible) channel having a slope of 0.0016 and $n = 0.025$. Proportion the section dimensions.

- **SOLUTION**: $Q = 11.5$ m$^3$/s  $S_0 = 0.0016$  $n=0.025$

\[ Q = \frac{A}{n} R^{2/3} S_0^{1/2} \]

\[ Z = \frac{n * Q}{\sqrt{S_0}} = A R^{2/3} = \left[ \frac{(b + zy)y}{b + 2y\sqrt{1 + z^2}} \right]^{5/3} \left[ b + 2y\sqrt{1 + z^2} \right]^{2/3} \]

- Assume $b = 6m$ and $z = 2$,
- Solve for $y = 1.04$ m
- (by trial an error)
- $Q = 11.5 \text{ m}^3/\text{s}$  $S_0 = 0.0016$  $n=0.025$
- $b = 6\text{ m}$ and $z= 2$,  $y = 1.04\text{ m}$
- For given $Q$ from DSİ’s curve

- Height of lining above water surface 0.33m
- Height of bank above water surface 0.63m

- Check stability :
- At critical flow $y_c = 0.692\text{ m}$
- For $y_n$  $Fr = 0.48$ subcritical and within limits
- Velocity $V = Q/A = 1.37\text{ m }/\text{s}$
- OK for sedimentation.

$$F_r^2 = \frac{Q^2T}{gA^3} = 1$$

$$F_r = \frac{V}{\sqrt{gD}}$$

$$F_r = \frac{V}{\sqrt{g(A/T)}}$$
Open Channel Design Example 1b

- A trapezoidal channel carrying 11.5 m³/s clear water is built with concrete (nonerodible) channel having a slope of 0.0016 and n= 0.025. Proportion the section dimensions. Use experience curve and z=1.5

- SOLUTION : Q = 11.5 m³/s  S₀ = 0.0016 n=0.025

\[
\left[ \frac{(b + zy)y^{5/3}}{b + 2y\sqrt{1 + z^2}} \right]^{2/3} = 7.1875
\]

- Take b = 2.5m,
- Solve for y = 1.56m
- (by trial an error)
- $Q = 11.5 \text{ m}^3/\text{s}$  
  $S_0 = 0.0016$  
  $n=0.025$
- $b = 2.5\text{m}$ and $z= 1.5, \ y = 1.56\text{m}$

- Check stability :
  - At critical flow $y_c = 0.692\text{m}$
  - For $y_n$  $Fr = 0.47$  subcritical and within limits
  - Velocity $V = Q/A = 1.52 \text{ m} /\text{s}$
  - OK for sedimentation.
Open Channel Design Example 1c

- A trapezoidal channel carrying 11.5 m³/s clear water is built with concrete (nonerodible) channel having a slope of 0.0016 and n= 0.025. Proportion the section dimensions. **Use best hydraulic section approach!**

- **SOLUTION :** $Q = 11.5 \text{ m}^3/\text{s}$  \( S_0 = 0.0016 \)  \( n=0.025 \)

  ![Diagram of trapezoidal channel]

  Best Hydraulic Section for Trapezoidal Channel

  \[ z = \frac{1}{\sqrt{3}} \quad b = \frac{2}{3} \sqrt{3} y \]

  \[ T = \frac{4}{3} \sqrt{3} y \]

  \[ A = \sqrt{3} y^2 \quad R = \frac{y}{2} \]

- Solve for $y = 2.03 \text{ m}$
- (by trial an error)
- $Q = 11.5 \text{ m}^3/\text{s}$  $S_0 = 0.0016$  $n = 0.025$
- $b = 2.34\text{ m}$  $y = 2.03\text{ m}$

- Check stability:
- At critical flow $y_c = 0.692\text{ m}$
- For $y_n$  $Fr = 0.38$  subcritical and within limits
- Velocity $V = Q/A = 1.49 \text{ m} / \text{s}$
- OK for sedimentation.