You have 90 minutes to complete the exam. Please do not leave the examination room in the first 30 minutes of the exam. There are six questions, of varying credit (100 points total). Indicate clearly your final answer to each question. You are allowed to use a calculator. During the exam, please turn off your cell phone(s). You cannot use the book or your notes. You have one page for “cheat-sheet” notes at the end of the exam papers. The answer key to this exam will be posted on Department of Mathematics and Computer Science board after the exam.

Good luck!

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Question 1. There are \( n \) married couple in a party. All the participants shake each other’s hands only once except his/her partner. What is the total number of handshakes at the party?

\[ \text{Answer.} \]

Question 2. A company decided to choose 6 of its employees by drawing and give them a weekend holiday as a gift for every weekend during one year.

(a) What should be the minimum number of employees of this company if all holiday groups are different then each other?

\[ \text{Answer.} \]

(b) It is given that the number of the employees of this company is equal to the minimum number that you find in part (a). Also we know that two brothers are working for this company. What is the probability of selecting their names consecutively in the first drawing?

\[ \text{Answer.} \]
(a) Find the coefficient of $\frac{1}{x^4}$ in the expansion of $\left(\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{3x}}\right)^7$.

*Answer:*

(b) In a group of 6 married couple, 4 people are selected at random. What is the probability that NOT married couple is selected?

*Answer:*
Question 4.
Show that if events $A$ and $B$ are independent then events $A'$ and $B'$ are independent.

Answer.

Question 5.
A continuous random variable $X$ has the following probability density function

$$f(x) = \begin{cases} 
  kx^{-4}, & x > 1, \\
  0, & \text{elsewhere}.
\end{cases}$$

(a) Find $k$.
Answer.

(b) Find the distribution function of the random variable $X$.
Answer.
Suppose that 3 calculators are randomly chosen without replacement from the following group of 10 calculators: 7 new, 1 used (working) and 2 out of order (not working). Let $X$ denote the number of new calculators chosen and $Y$ denote the number of used calculators chosen.

(a) Find the joint probability distribution table.

Answer.

(b) Find the conditional distribution of $Y$ given $X = 2$.

Answer.

(c) Determine whether or not $X$ and $Y$ are independent.

Answer.