You have 90 minutes to complete the exam. Please do not leave the examination room in the first 30 minutes of the exam. There are six questions, of varying credit (100 points total). Indicate clearly your final answer to each question. You are allowed to use a calculator. During the exam, please turn off your cell phone(s). You cannot use the book or your notes. You have two pages for “cheat-sheet” notes. The answer key to this exam will be posted on Department of Mathematics and Computer Science board after the exam.

Good luck!

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|-------------|-------------|-------------|-------------|-------------|-------------|-------|
Suppose that a bag has 3 boxes, one of them containing 2 bullets and two of them containing 1 bullet. A person is asked to randomly choose a box and then shoot a target until the last bullet is fired. Suppose 90% for this person, the bullet will hit the target. Let $X$ be the number of bullets hitting the target and $Y$ be the number of bullets in the selected box that the person has.

(a) Find the joint probability distribution of $X$ and $Y$.

*Answer.*

(b) Find the conditional variance of $X$ given $Y = 1$.

*Answer.*
(a) The moment-generating function of the random variable $X$ is $M_X(t)$. If we let

$$R_X(t) = \ln M_X(t),$$

show that $\frac{d}{dt} R_X(t) \bigg|_{t=0} = \mu$ and $\frac{d^2}{dt^2} R_X(t) \bigg|_{t=0} = \sigma^2$.

Answer.

(b) Use the results given in (a) to find the mean and the variance of $X$ having the moment-generating function $M_X(t) = e^{4(e^t-1)}$.

Answer.
Question 3.  

A random variable $X$ has the density function given by 

$$f(x) = \begin{cases} 
6x(1-x) & \text{for } 0 < x < 1 \\
0 & \text{elsewhere.}
\end{cases}$$

(a) Find $P(|X - \mu| \geq 0.7)$.

Answer.

(b) Find an upper bound for $P(|X - 0.5| \geq 0.7)$ and compare it with the result in (a).

Answer.
Question 4.  
10 + 10 points

A doctor wishes to perform an experiment on 5 patients. Suppose that the probability that a randomly selected patient agrees to participate to this experiment is 0.2.

(a) What is the probability that 15 patients must be asked before 5 are found who agree to participate?

Answer.

(b) What is the probability that at least 4 patients must be asked before the first patient is found who agree to participate?

Answer.

Question 5.  
15 points

Suppose you roll a pair of non-fair, six sided dice 144 times, which are loaded in such a way that each odd number is three times as likely to occur as each even number. Let the random variable $X$ denote the sum of the points on both dice which can come up in one roll. Use a Poisson approximation to determine the probability of observing that $X = 6$ exactly four times in this 144 rolls.

Answer.
Suppose that there are two types of airplanes, one of them has two engines and other one has four engines. It is given that an airplane engine will fail, during the flight, with probability \(1 - \theta\) (where \(\theta \neq 1\)), independently from engine to engine; and suppose that the airplane will make a successful flight if at least 50 percent of its engines remain operative. For what values of \(\theta\) is a two-engine plane preferable to a four-engine plane?

\textit{Answer.}