Final Exam

Fall 2013-2014

Number:

Name:

Department:

Section:

Directions – You have 90 minutes to complete the exam. Please do not leave the examination room in the first 30 minutes of the exam. There are six questions, of varying credit (100 points total). Indicate clearly your final answer to each question. You are allowed to use a calculator. During the exam, please turn off your cell phone(s). You cannot use the book or your notes.

Good luck!

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Question 1.  

An automobile dealer has kept records on the customers who visited his showroom. 45% of the people who visited his dealership were female. Furthermore, his records show that 32% of the females who visited his dealership purchased an automobile, while 24% of the males who visited his dealership purchased an automobile.

(a) What is the probability that a customer entering the showroom will buy an automobile? 

Answer. Define the events: $F =$ Female customer, $M =$ Male customer, $B =$ customer purchases an automobile, $B^C =$ customer does not purchase an automobile. So,

$$P(B) = P(F)P(B|F) + P(M)P(B|M) = 0.45 \times 0.32 + 0.55 \times 0.24 = 0.276$$

(b) Suppose a customer visited the showroom but did not purchased a car. What is the probability that the customer was male? 

Answer. We use Bayes’ Theorem

$$P(M|B^C) = \frac{P(M)P(B^C|M)}{P(M)P(B^C|M) + P(F)P(B^C|F)} = \frac{P(M \cap B^C)}{P(M \cap B^C) + P(F \cap B^C)}$$

$$= \frac{0.55 \times 0.76}{0.55 \times 0.76 + 0.45 \times 0.68} = \frac{209}{362} \approx 0.577$$

Question 2.  

The rector of a large university wishes to estimate the average age of the students presently enrolled. From past studies, the standard deviation is known to be 2 years. A sample of 196 students is selected, and the mean is found to be 20.2 years. Find the 97% confidence interval of the population mean. 

Answer. Substituting $n = 196, \bar{x} = 20.2, \sigma = 2,$ and $z_{0.015} = 2.17$ into the confidence interval formula, we get

$$20.2 - 2.17 \times \frac{2}{\sqrt{196}} < \mu < 20.2 + 2.17 \times \frac{2}{\sqrt{196}}$$

or

$$20.2 - 0.31 < \mu < 20.2 + 0.31$$

which reduces to

$$19.89 < \mu < 20.51.$$
Question 3. 10 + 10 points
A Company gives each of its employees a knowledge test. The scores on the test are normally distributed with a mean of 70 and a standard deviation of 20. A simple random sample of 16 is taken from a population of 420.

(a) What is the probability that the average knowledge test score in the sample will be between 65.15 and 72.15?
Answer. Since \( \mu_X = 70 \) and \( \sigma = \frac{20}{\sqrt{16}} = 5 \) we have
\[
P(65.15 \leq X \leq 72.15) = P \left( \frac{65.15 - 70}{5} \leq Z \leq \frac{72.15 - 70}{5} \right) = P(-0.97 \leq Z \leq 0.43)
\]
\[
= 0.3340 + 0.1664 = 0.5004.
\]

(b) Find a value, \( K \), such that \( P(X \geq K) = 0.0073 \).
Answer. It is given that
\[
P(X \geq K) = P(Z \geq \frac{K - 70}{5}) = 0.0073.
\]
Since \( X \geq K \) we have \( 0.5 - 0.0073 = 0.4927 \) and corresponding \( z \) value is 2.44. Thus,
\[
\frac{K - 70}{5} = 2.44 \Rightarrow K = 82.2.
\]

Question 4. 10 + 10 points
The following table shows the number of hours per day of watching TV in a sample of 285 people

<table>
<thead>
<tr>
<th>Hours</th>
<th>0 ≤ h &lt; 3</th>
<th>3 ≤ h &lt; 6</th>
<th>6 ≤ h &lt; 9</th>
<th>9 ≤ h &lt; 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>91</td>
<td>48</td>
<td>112</td>
<td>34</td>
</tr>
</tbody>
</table>

(a) What is the mean number of TV viewing hours in this group?
Answer.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Hours} & \text{Frequency} & \text{Mid-point} & \text{Mid-point } \times \text{ Frequency} \\
\hline
0 \leq h < 3 & 91 & 1.5 & 136.5 \\
3 \leq h < 6 & 48 & 4.5 & 216 \\
6 \leq h < 9 & 112 & 7.5 & 840 \\
9 \leq h < 12 & 34 & 10.5 & 357 \\
\hline
\text{TOTAL} & & & 1549.5 \\
\hline
\end{array}
\]

\[
\text{Mean} = \frac{1549.5}{285} = 5.437 \text{ hours}
\]

(b) What is the median number of TV viewing hours?
Answer. The median is the 143rd values. In this case it lies in the 6 ≤ h < 9 class interval. The 4th value in this interval is needed. It is estimated as
\[
\text{Median} = 6 + \frac{9 - 6}{112} \times 4 = 6.107 \text{ hours}
\]
Question 5.  

Four players are playing a game with a six-sided fair die. First, Player 1 rolls the die. After the first roll, the Player 2 rolls the die, and then Player 3, and then Player 4. After that Player 1 rolls the die again and then Player 2 and so on. This is repeated until a six appears, at which time the game is stopped. A player wins with the number six. Compute the probability Player 3 wins the game.

*Answer.*

\[
\left( \frac{5}{6} \right)^2 \frac{1}{6} + \left( \frac{5}{6} \right)^6 \frac{1}{6} + \left( \frac{5}{6} \right)^{10} \frac{1}{6} + \cdots = \left( \frac{5}{6} \right)^2 \frac{1}{6} \left( 1 + \left( \frac{5}{6} \right)^4 + \left( \frac{5}{6} \right)^8 + \left( \frac{5}{6} \right)^{12} + \cdots \right) \\
= \left( \frac{5^2}{6^3} \right) \left( \frac{1}{1 - \frac{5^4}{6^4}} \right) = \left( \frac{5^2}{6^3} \right) \left( \frac{6^4}{6^4 - 5^4} \right) \\
= 0.2235
\]

Question 6.  

Suppose that you have a bag filled with 150 marbles of which 10% are green. Find the probability that no more than 3 green marbles will be chosen in a sample size of 7 by using

(a) the formula of the hypergeometric distribution. (Give your result as a formula, do not evaluate it)

*Answer.* Substituting \( N = 150, M = 150 \times 0.10 = 15, n = 7 \) and \( x \leq 3 \) into the formula for hypergeometric distribution, we get

\[
h(x \leq 3; 7, 150, 15) = \binom{15}{0} \binom{135}{7} + \binom{15}{1} \binom{135}{6} + \binom{15}{2} \binom{135}{5} + \binom{15}{3} \binom{135}{4}
\]

(b) the binomial distribution as an approximation. (Give your result as a formula, do not evaluate it)

*Answer.*

\[
b(x \leq 3; 7; 0.1) = \binom{7}{0} (0.1)^0 (0.9)^7 + \binom{7}{1} (0.1)^1 (0.9)^6 + \binom{7}{2} (0.1)^2 (0.9)^5 + \binom{7}{3} (0.1)^3 (0.9)^4
\]