İstanbul Kültür University

MCB1007
Introduction to Probability and Statistics

First Midterm

Fall 2014-2015

Number:
Name:
Department:

Directions — You have 90 minutes to complete the exam. Please do not leave the examination room in the first 30 minutes of the exam. There are six questions, of varying credit (100 points total). Indicate clearly your final answer to each question. You are allowed to use a calculator. During the exam, please turn off your cell phone(s). You cannot use the book or your notes. You have one page for “cheat-sheet” notes at the end of the exam papers. The answer key to this exam will be posted on Department of Mathematics and Computer Science board after the exam.

Good luck!

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Question 1.
Question 2.
Question 3.

Question 4.
Question 5.
Question 6.

TOTAL
Question 1.  
(a) How many odd positive integers less than 300 can be formed using the digits 0, 1, 2, 3, and 4, without repetition?  
Answer.

(b) In İstanbul city, all vehicle license plates have 2 letters from the 23 letters of the alphabet followed by 4 one-digit numbers from 0, 1, 2, ⋯, 9. How many different license plates are possible for İstanbul if repetition is allowed?  
Answer.

Question 2.  
(a) An urn contains six balls numbered 1 through 6. Three balls are randomly drawn from the urn in succession, without replacement. What is the probability that the smallest number in this sampling equal to 2?  
Answer.

(b) Find the constant term in the expansion of \( \left( \frac{1}{x^2} - 2x^3 \right)^{10} \).  
Answer.
The density function of the continuous random variable $X$ is given by

$$f(x) = \begin{cases} 
  c(x + \sqrt{x}), & \text{for } 0 < x < 1, \\
  0, & \text{otherwise.} 
\end{cases}$$

(a) What is the constant $c$?

Answer.

(b) Find the distribution function of the random variable $X$.

Answer.
Factories A, B and C produce a textile product. Factory A produces 2 times as many textile products as Factory B and Factory C. Factory A and Factory B produce defective products 5% of the time and Factory C produces defective products 1% of the time. A textile product is selected at random and it is found to be defective. What is the probability it came from Factory C?

Answer.

Question 5.

Suppose that a couple will continue having children until have a boy, under the assumption that they have ability to have children and theoretically number of births goes to infinity. So, if they have a female child they keep having more children until they have a boy. If they have a boy, they stop having children. Let $X$ be the number of births. Assume that outcomes of births are independent of each other, and boys and girls are equally likely.

(a) Find the probability distribution of the random variable $X$?

Answer.

(b) Verify that the function in (a) can serve as the probability distribution of $X$.

Answer.
2 balls are selected at a random from an urn without replacement containing 3 blue and 5 white balls. Let the random variable $X$ is the number of blue balls in the first draw and $Y$ is the number of white balls in the second draw.

(a) Find the joint probability distribution as a table.

*Answer.*

(b) Find the conditional distribution of $Y$ given $X = 1$.

*Answer.*

(c) Determine whether or not $X$ and $Y$ are independent.

*Answer.*