MAT 003 - INTRODUCTION TO PROBABILITY THEORY AND STATISTICS
Fall 2010

Second Midterm

December 14, 2010

Number: Name:

Directions: You have 90 minutes to complete the exam. Please do not leave the examination room in the first 30 minutes of the exam. There are four questions, of varying credit (100 points total). You must show your working to get full marks for a question, an answer alone will not earn credit. Indicate clearly your final answer to each question. You are not allowed to use a calculator. During the exam, please turn off your cell phone(s). You cannot use the book or your notes. You have one double-sided page for “cheat-sheet” notes at the end of the exam papers. The answer key to this exam will be posted on Department of Mathematics and Computer Science board after the exam.

Good luck!

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Problem 1.  

The joint density function of $X$ and $Y$ is given by

$$f(x, y) = \begin{cases} 
3(x + y), & 0 \leq x + y \leq 1, \ 0 \leq x, \ y < \infty \\
0, & \text{otherwise},
\end{cases}$$

(a) Are $X$ and $Y$ independent?  
   Answer.

(b) Find the conditional density function of $Y$ given $X = 1/2$.  
   Answer.
Problem 2.  

Suppose that $X$ and $Y$ are discrete random variables with values 1, 2, 3 and 4 and the joint probability distribution is given by

$$f(x, y) = \begin{cases} 
\frac{1}{16}, & x = y \\
\frac{2}{16}, & x < y \\
0, & x > y.
\end{cases}$$

(a) Find two formulas for marginal probability functions $f_1(x)$ and $f_2(y)$ of $X$ and $Y$, respectively.

**Answer.**

(b) Find the conditional expectation of $Y$ given $X = 3$.

**Answer.**

(c) Find the conditional variance of $Y$ given $X = 3$.

**Answer.**
Problem 3. 7 + 5 + 5 points

If a random variable $X$ is such that $E[(X - 1)^2] = 10$, $E[(X - 2)^2] = 6$,

(a) Find the mean of $X$.
   Answer.

(b) Find the standard deviation of $X$.
   Answer.

(c) Find an upper bound for $P\left(\left|X - \frac{7}{2}\right| \geq \sqrt{13}\right)$.
   Answer.
Let $X$ be a random variable having the probability density function

$$f(x) = \begin{cases} e^{-1-x}, & -1 < x \\ 0, & \text{otherwise}. \end{cases}$$

(a) Find the moment generating function of $X$.

Answer.

(b) Find the first three moments about the origin by using (a).

Answer.

(c) Find the first three moments about the mean by using (b).

Answer.