CHAPTER 7: ENGINEERING ECONOMICS

The aim is to think about and understand the power of money on decision making

APPLICATION AREA

- PRESENT ECONOMY
- PERSONAL FINANCE
- ENGINEERING PROJECTS
PRESENT ECONOMY

- When total cost equals to $6000
- To make $10,000 monthly profit at the monthly sales of 1000 units
- What must the unit selling price be

Revenue

Profit

Total Cost

Solve for price?

Revenue = sales x price

Profit = Revenue − Total Cost

Price = \frac{10,000 + 6000}{1000} = \$16 / unit
PERSONAL FINANCE

☐ A bank account earns 20% annual interest rate
☐ To have $10,000 savings 3 years from now
☐ How much money should one put into that bank account?

Discount by 20%

<table>
<thead>
<tr>
<th>Today</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5,787</td>
<td>$6,944</td>
<td>$8,333</td>
<td>$10,000</td>
</tr>
</tbody>
</table>

$5,787 \times (1+0.2) = 6,944$

$6,944 \times (1+0.2) = 8,333$

$8,333 \times (1+0.2) = 10,000$
ENGINEERING PROJECTS

- How much money can investor afford to spend to implement a new engineering design?
- that will make annual uniform saving of $2000 for five years
- if 20% rate of return on investment is desired

Discount incomes at 20%

<table>
<thead>
<tr>
<th>Year</th>
<th>Saving (income)</th>
<th>Discount factor</th>
<th>Equivalent amount in Year 0 (Today)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2000</td>
<td>(1+0.2)^-1</td>
<td>1666.67</td>
</tr>
<tr>
<td>2</td>
<td>2000</td>
<td>(1+0.2)^-2</td>
<td>1388.89</td>
</tr>
<tr>
<td>3</td>
<td>2000</td>
<td>(1+0.2)^-3</td>
<td>1157.41</td>
</tr>
<tr>
<td>4</td>
<td>2000</td>
<td>(1+0.2)^-4</td>
<td>964.51</td>
</tr>
<tr>
<td>5</td>
<td>2000</td>
<td>(1+0.2)^-5</td>
<td>803.76</td>
</tr>
</tbody>
</table>

Σ = 5981.22
Discounted incomes at 20% should be equal to investment

If required investment > 5981.22
Project does not earn 20% and rejected

If required investment = 5981.22
Project earns 20% and accepted

BREAKEVEN ANALYSIS

- Breakeven point method deals with the effect of alternative rates of operation on revenues and costs.
- Breakeven point method is used within a specified period during which the essential nature of situation cannot be changed.
- The term of breakeven is used to denote the output rate at which total cost and total revenue are equal in the considered period.
**Notation**

- Q = DEMAND or QUANTITY produced
- \( p = \text{PRICE per unit} \)
- \( v = \text{VARIABLE cost per unit} \)
- \( TR = \text{total REVENUE} = p \times Q \)
- \( CV = \text{VARIABLE cost} = v \times Q \)
- CF = FIXED cost
- \( CT = \text{total COST} = CF + CV \)

**Breakeven Point**

![Breakeven Point Graph](image)

- \( TR = p \times Q \)
- \( CT = CF + v \times Q \)
- \( CV = v \times Q \)
- \( TR = CT \) (no loss; no profit)
- Q\( _{BE} \) (Breakeven point)
- FIXED COST
- LOSS

Cost and Revenue, $
Breakeven Point

- at BREAKEVEN point \( (Q_{BE}) \)
  \[ TR = CT; \text{ so NO LOSS & NO PROFIT} \]

- Solve for \( Q \)?
  \[ p \times Q = CF + v \times Q \]
  \[ Q_{BE} = \frac{CF}{p - v} \]

Breakeven Example

- An engineering consulting firm measures its output in a standard service hour unit. The variable cost \( (v) \) is $62 per standard service hour. The charge-out rate $85.56 per hour. The maximum output of the firm is 160,000 hours per year, and its fixed cost \( (CF) \) is $2,024,000 per year. What is the breakeven point in standard service hours and in percentage of total capacity?
Breakeven Point

Cost and Revenue, $
TR = 85.56 \times Q$
$CT = 2,024,000 + 62 \times Q$

Solve for breakeven point?

$Q_{BE} = \frac{CF}{p-v} = \frac{2,024,000}{85.56 \text{ / hour} - 62 \text{ / hour}} = 85,908 \text{ hours}
$

Firm’s breakeven service hours

$Q_{BE \%} = \frac{Q_{BE}}{\text{total CAPACITY}} = \frac{85,908 \text{ hours}}{160,000 \text{ hours}} = 0.54 = 54\%$

Firm’s breakeven is reached at 54 % of capacity
Breakeven Point

TR = CT (no loss; no profit)
CT = $2,024,000 + 62 x Q
TR = 85.56 x Q
Q_{BE} = 85,908 hours

Time value of money

- Time value of money is designated by interest factors in engineering economy studies
- While moving to the future, money today is compounded and will be more worthy because of the interest rate
- When moving back to now, money in the future is discounted and will be less worthy
**Notation**

- $i$ = annual interest rate (%)
- $N$ = number of annual interest periods (e.g., years)
- $P$ = present principal sum
  (e.g., $1500 at the end of year 0, P = $1500)
- $A$ = single payment in a series of $n$ equal payments made
  at the end of each annual interest period starting at the end of the first year
  (e.g., $100 at the end of each year for 5 years, $A = $100)
- $F$ = future sum, $n$ annual periods hence, equal to the compound amount of a present sum $P$, or equal to the sum of the compound amounts of payments $A$, in a series
  (e.g., $2000 10$ years from now, $F = $2000)

**P, F and A on cash flow diagram**

![Cash Flow Diagram](image-url)

- Year 1
- End of year 1
- $P$ at year 0
- $F$ at year $N$
- Payments $A$ at years 1 through $N-1$
## Find F given P

<table>
<thead>
<tr>
<th>Year</th>
<th>Investment at the Beginning of Year</th>
<th>Interest Charged (i%)</th>
<th>Amount at the End of Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P</td>
<td>i x P</td>
<td>P(1+i)</td>
</tr>
<tr>
<td>2</td>
<td>P(1+i)</td>
<td>i x [P(1+i)]</td>
<td>P(1+i)²</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>N</td>
<td>P(1+i)⁻¹</td>
<td>i x [P(1+i)⁻¹]</td>
<td>P(1+i)ᴺ</td>
</tr>
</tbody>
</table>

\[ F = P(1+i)^N \]

## Symbolic interest factors

- \((1+i)^N = (F/P, \ i\%, \ N)\)
- Example: \(i = 20\%, \ N = 5 \text{ years}\)
  \[(F/P, 20\%, 5) = (1 + 0.2)^5 = 2.4883\]
  Suppose \(P = $1000, \ F = ?\)
  \[F = 1000(F/P, 20\%, 5) = $2488.3\]
### Interest tables

<table>
<thead>
<tr>
<th>(F/P, i%, N)</th>
<th>i% interest rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>N (periods)</td>
<td>1%</td>
</tr>
<tr>
<td>1</td>
<td>1.0100</td>
</tr>
<tr>
<td>2</td>
<td>1.0201</td>
</tr>
<tr>
<td>3</td>
<td>1.0303</td>
</tr>
<tr>
<td>4</td>
<td>1.0406</td>
</tr>
<tr>
<td>5</td>
<td>1.0510</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>10</td>
<td>1.1046</td>
</tr>
</tbody>
</table>

(F/P, 20%, 5) = 2.4883

---

### Interest formulas

- **Find P given F:**
  \[
P = F \left(1+i\right)^{-N} = F \left(P/F, \ i\%, \ N\right)
  \]

- **Find F given A:**
  \[
  F = A \left[\frac{(1+i)^N - 1}{i}\right] = A \left(F/A, \ i\%, \ N\right)
  \]

- **Find A given F:**
  \[
  A = F \left[\frac{i}{(1+i)^N - 1}\right] = F \left(A/F, \ i\%, \ N\right)
  \]
Interest formulas

- **Find P given A:**
  \[ P = A \left( \frac{(1+i)^N - 1}{i(1+i)^N} \right) = A \left( P/A, \ i\%, \ N \right) \]

- **Find A given P:**
  \[ A = P \left( \frac{i(1+i)^N}{(1+i)^N - 1} \right) = P \left( A/P, \ i\%, \ N \right) \]

Excel formulas for interest factors

For \((A/P,i\%,N)\) and \((A/F,i\%,N)\)

\[ = \text{pmt}(\text{rate} ; \text{nper} ; \text{pv} ; [\text{fv}] ; [\text{type}]) \]

For \((P/A,i\%,N)\) and \((P/F,i\%,N)\)

\[ = \text{pv}(\text{rate} ; \text{nper} ; \text{pmt} ; [\text{fv}] ; [\text{type}]) \]

For \((F/A,i\%,N)\) and \((F/P,i\%,N)\)

\[ = \text{fv}(\text{rate} ; \text{nper} ; \text{pmt} ; [\text{pv}] ; [\text{type}]) \]

- green = given P  
- purple = given F  
- black = given A
Some examples:

\[ \text{pmt}(0,2;5;-1;0) = (A/P,20\%,5) = 0.3344 \]
\[ \text{pmt}(0,1;15;0;-1) = (A/F,10\%,15) = 0.0315 \]
\[ \text{pv}(0,15;10;-1;0) = (P/A;15\%;10) = 5.0188 \]
\[ \text{pv}(0,25;20;0;-1) = (P/F;25\%;20) = 0.0115 \]
\[ \text{fv}(0,15;10;-1;0) = (F/A;15\%;10) = 20.3037 \]
\[ \text{fv}(0,15;10;0;-1) = (F/P;15\%;10) = 4.0456 \]

Example problem for finding F given P (i = 10%)

A firm borrows $1000 for eight years.

\[ P = 1000 \]  How much must it repay in a lump sum at the end of eighth year?

Solution:
\[ F = P (F/P, 10\%, 8) = 1000 (2.1436) \]
\[ = 2143.6 \]  F=?
Example problem for finding P given F (i = 10%)

- A firm wishes to have $2143.6 eight years from now. $F = 2143.6$
- What amount should be deposited now to provide this?

**Solution:**

$$P = F \times (P/F, 10\%, 8) = 2143.6 \times 0.4665$$

$$= \$1000$$

Example problem for finding F given A (i = 10%)

- If eight annual deposits of $187.45 each are placed in an account $F=?$
- How much money has accumulated immediately after the last deposit?

**Solution:**

$$A = 187.45$$

$$F = A \times (F/A, 10\%, 8) = 187.45 \times 11.4359$$

$$= \$2143.6$$
Example problem for finding $P$ given $A$ (i = 10%)

- To provide for eight end-of-year withdrawals $187.45
- How much should be deposited in a fund now?

\[ A = 187.45 \]

Solution:
\[ P = A \left( \frac{P}{A}, 10\%, 8 \right) = 187.45(5.3349) \]
\[ = $1000 \]

Example problem for finding $A$ given $F$ (i = 10%)

- To accumulate $2,143.6 at the time of the eighth annual deposit
- What uniform annual amount should be deposited each year?

\[ F = 2,143.6 \]

Solution:
\[ A = F \left( \frac{A}{F}, 10\%, 8 \right) = 2,143.6(0.0874) \]
\[ = $187.45 \]
Example problem for finding A given P (i = 10%)

P=?
- To repay a loan of $1000
- What is the size of eight equal annual payments?

0 1 2 3 4 5 6 7 8

Solution:

A = ?

A = P (A/P, 10%, 8) = 1000(0.18745)
= $187.45

Investment appraisal methods

- **Present Worth (PW):** present equivalent amount of all cash flows associated with an investment project
- **Internal Rate of Return (IRR):** interest rate that an investment project earns
Cash flow notation

- **S** = salvage value
- **R** = annual revenue
- **E** = annual expenses
- **I** = investment

Present Worth (PW)

- Compute the present equivalent of the estimated cash flows using the MARR as the interest rate.
- If **PW** (MARR) \( \geq 0 \), then the project is profitable.
- If **PW** (MARR) < 0, then the project is not profitable.
Present Worth (PW)

\[ PW = - I + (R - E)(P/A, i\%, N) + S(P/F, i\%, N) \]

\[ i\% = MARR = \text{minimum attractive rate of return} \]

Example Problem for PW

Cost/Revenue Estimates

- Initial Investment $50,000
- Annual Revenues 20,000
- Annual Operating Costs 2,500
- Salvage Value @ EOY 5 10,000
- Study Period 5 years
- MARR 20% /year
Cash flow diagram

N = 5 years
MARR=20%/yr.

R = 20,000
S = 10,000
E = 2,500
I = 50,000

PW at MARR=20%

PW(20%) = - 50,000 + (20,000 - 2,500)(P/A,20%,5) + 10,000(P/F,20%,5)
= - 50,000 + 17,500(2.9906) + 10,000(0.4019)
= - 50,000 + 52,335.5 + 4019 = $6,354.5
The result of PW tells us:

- Since PW(20%) = 6,354.5, the project is accepted
- We have recovered our entire $50,000 investment
- We have earned our desired 20% on this investment
- We have made a lump sum equivalent profit of $6,354.50 beyond what was expected (required)

Internal Rate of Return (IRR)

- Compute the interest rate ($i'$) that makes the PW of a project's estimated cash flows equal to zero
- If (IRR = $i'$) ≥ MARR, project is accepted
- If (IRR = $i'$) < MARR, project is rejected
Find \( i'\% \) such that the \( PW(i'\%) = 0 \)

\[ 0 = -50,000 + 17,500(P/A,i'\%,5) + 10,000(P/F,i'\%,5) \]

Find \( i'\% \) by Trial and Error

For \( i' = 20\% \)

\[
PW = -50,000 + 17,500(P/A,20\%,5) + 10,000(P/F,20\%,5)
\]

\[ = 6354.50 \text{ tells us that } i' > 20\% \]

For \( i' = 25\% \)

\[
PW = -50,000 + 17,500(P/A,25\%,5) + 10,000(P/F,25\%,5)
\]

\[ = 339.75 \text{ tells us that } i' > 25\% \]

For \( i' = 30\% \)

\[
PW = -50,000 + 17,500(P/A,30\%,5) + 10,000(P/F,30\%,5)
\]

\[ = -4,684.24 \text{ tells us that } i' < 30\% \]
### Find exact value by Linear Interpolation

<table>
<thead>
<tr>
<th>i%</th>
<th>PW</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) 25</td>
<td>+ 339.75  (f)</td>
</tr>
<tr>
<td>(b) i'</td>
<td>0         (g)</td>
</tr>
<tr>
<td>(c) 30</td>
<td>- 4684.24 (h)</td>
</tr>
</tbody>
</table>

#### Calculation:

\[
\frac{i' - 25}{5} = \frac{339.75}{5023.99} = \frac{30 - 25}{5}
\]
Find exact value by Linear Interpolation

\[ i' = IRR = 25 + \frac{339.75}{5023.99} (5) = 25.3\% \]

The result of IRR tells us:

- Since (IRR = 25.3\%) > (MARR = 20\%), the project is accepted
- 25.3\% return on investment is obtained
- The project earns 5.3\% more than what was desired per year