Course Objectives

- Understand cost concepts and how to make an economic decision with present economy studies
- Discuss time value of money for personal finance issues
- Learn methods for determining whether an investment project is acceptable (profitable) or not, and how to compare two or more investment alternatives using these methods
- Consider involving income-tax, inflation rate and uncertainty with engineering economy studies
- Finally, develop spreadsheet models as problem solver for engineering economy studies.
Course Materials

**Text Book:**
William G. Sullivan, Wicks and Koelling
Prentice Hall; 14 edition (May 13, 2008)

**Software:**
Excel templates for spreadsheet modelling

**Lecture notes:**
Power point slides (to be published via course web page)

Course Topics

**Chapter 1:** (Chapter 2 in the Text Book)
Cost concepts and present economy studies

**Chapter 2:** (Chapter 4 in the Text Book)
Time value of money

**Chapter 3:** (Chapters 5&10 in the Text Book)
Investment appraisal (evaluating a single project)
Course Topics

**Chapter 4:** (Chapters 6&10 in the Text Book)
Comparison among alternatives

**Chapter 5:** (Chapter 7 in the Text Book)
Depreciation and income taxes

**Chapter 6:** (Chapter 8 in the Text Book)
Price change and inflation rate

**Chapter 7:** (Chapters 11&12 in the Text Book)
Dealing with uncertainty

Grading

- **Exams** (85% of total grade)
  - Quizzes (10%)
  - Midterm I and Midterm II (40%)
  - Final (35%)

- **Assignments** (10% of total grade)

- **Participation** (5% of total grade)
COST CONCEPTS

- **Fixed / Variable Costs** – If costs change appreciably with fluctuations in business activity, they are “variable.” Otherwise, they are “fixed.”

- **A widely used cost model is:**
  \[
  \text{Total Costs} = \text{Fixed Costs} + \text{Variable Costs}
  \]

- **Some examples of fixed costs:** Insurance, taxes on facilities, administrative salaries, rental payments and initial setup or installation.

- **Some examples of variable costs:** direct labor, direct material, unit transportation.
Example 1.1

- In connection with surfacing a new highway, a contractor has a choice of two sites on which to set up the mixing plant equipment.
- The contractor estimates that it will cost $1.15 per cubic meter per kilometer to haul the asphalt paving material from the mixing plant to the job site.
- If site B is selected, there will be an added charge of $96 per day for a flagman.
- The job requires 50,000 cubic meters of mixed asphalt paving material. It is estimated that four months (17 weeks of five working days per week) will be required for the job.

Example 1.1 (continued)

<table>
<thead>
<tr>
<th>Cost Factors</th>
<th>Site A</th>
<th>Site B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average hauling distance</td>
<td>6 km</td>
<td>4.3 km</td>
</tr>
<tr>
<td>Monthly rental of site</td>
<td>$1000</td>
<td>$5000</td>
</tr>
<tr>
<td>Cost to setup and remove equipment</td>
<td>$15000</td>
<td>$25000</td>
</tr>
<tr>
<td>Hauling expense</td>
<td>$1.15 / m³/km</td>
<td>$1.15 / m³/km</td>
</tr>
</tbody>
</table>

a) Compare the two sites in terms of their fixed, variable, and total costs.
b) For the selected site, how much profit can be made if paid $8.05 per cubic meter delivered to the job site?
Example 1.1 (solution to a)

<table>
<thead>
<tr>
<th>Fixed Costs</th>
<th>Site A</th>
<th>Site B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly rental of site</td>
<td>$1000 x 4</td>
<td>$5000 x 4</td>
</tr>
<tr>
<td>Cost to setup and remove equipment</td>
<td>$15,000</td>
<td>$25,000</td>
</tr>
<tr>
<td>Flagman wage</td>
<td>0</td>
<td>$96 x 85</td>
</tr>
<tr>
<td><strong>Total Fixed Costs</strong></td>
<td><strong>$19,000</strong></td>
<td><strong>$53,160</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable Costs</th>
<th>Site A</th>
<th>Site B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hauling expense</td>
<td>$1.15 / m³/km × 6 km × 50,000 m³ = <strong>$345,000</strong></td>
<td>$1.15 / m³/km × 4.3 km × 50,000 m³ = <strong>$247,250</strong></td>
</tr>
<tr>
<td><strong>Total Costs</strong></td>
<td><strong>$364,000</strong></td>
<td><strong>$300,410</strong></td>
</tr>
</tbody>
</table>

Example 1.1 (solution to b)

\[
\text{PROFIT} = \text{REVENUE} - \text{TOTAL COST}
\]

\[
\text{REVENUE} = 8.05 / \text{m}^3 \times 50,000 \text{ m}^3 = 402,500
\]

\[
\text{PROFIT} = 402,500 - 300,410 = 102,090
\]
Example 1.2

Consider the following cost and production data and develop cost estimating relationship (CER) equation between produced units and total cost, and estimate the cost for production capacity of 2100 units.

<table>
<thead>
<tr>
<th>Month</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Produced units</td>
<td>1000</td>
<td>850</td>
<td>1500</td>
<td>900</td>
<td>450</td>
<td>690</td>
<td>1150</td>
</tr>
<tr>
<td>Total cost</td>
<td>7000</td>
<td>6550</td>
<td>8500</td>
<td>6700</td>
<td>5350</td>
<td>6070</td>
<td>7450</td>
</tr>
</tbody>
</table>

Least Squares Normal Equations

\[ y = a + bx \quad \text{CER} \]

- \( y \) = total cost
- \( a \) = fixed cost
- \( b \) = unit variable cost

\[
b = \frac{n \sum_{i=1}^{n} x_i y_i - \left( \sum_{i=1}^{n} x_i \right) \left( \sum_{i=1}^{n} y_i \right)}{n \sum_{i=1}^{n} x_i^2 - \left( \sum_{i=1}^{n} x_i \right)^2}
\]

\[
a = \frac{\sum_{i=1}^{n} y_i - b \left( \sum_{i=1}^{n} x_i \right)}{n}
\]
Example 1.2 (solution)

\[ n = 7 \]

\[
\sum_{i=1}^{n} x_i = 6540, \quad \sum_{i=1}^{n} y_i = 47,620, \quad \sum_{i=1}^{n} x_i^2 = 6,783,600, \quad \sum_{i=1}^{n} x_i y_i = 46,510,800.
\]

\[
b = \frac{7(46,510,800) - (6540)(47,620)}{7(6,783,600) - (6540)^2} = 3
\]

\[
a = \frac{(47,620) - 3(6540)}{7} = 4000
\]

\[ y = 4000 + 3x \]

\[ x \text{ (production)} = 2100 \text{ units} \]

\[ y \text{ (total cost)} = $4000 + ($3 / \text{unit})(2100 \text{ units}) = $10,300 \]

BREAKEVEN ANALYSIS

**Notation**

- \( TR \) = Total Revenue
- \( CT \) = Total Cost
- \( CF \) = Fixed Cost
- \( CV \) = Variable Cost
- \( v \) = Unit Variable Cost per unit
- \( p \) = Price per unit
- \( Q \) = Demand (output rate)
BREAKEVEN ANALYSIS

at BREAKEVEN point \( Q_{BE} \), \( TR = CT \),
so NO LOSS & NO PROFIT

\[ p(Q) = CF + v(Q), \]

\[ Q_{BE} = \frac{CF}{p-v}, \quad \text{for } p > v. \]

if \( DEMAND \ (Q) > Q_{BE} \), then PROFIT
if \( DEMAND \ (Q) < Q_{BE} \), then LOSS
Example 1.3

An engineering consulting firm measures its output in a standard service hour unit. The variable cost (v) is $62 per standard service hour. The charge-out rate (i.e., selling price “p”) is $85.56 per hour. The maximum output of the firm is 160,000 hours per year, and its fixed cost (CF) is $2,024,000 per year. For this firm,

a) What is the breakeven point in standard service hours and in percentage of total capacity?

b) What is the percentage reduction in the breakeven point (sensitivity) if fixed costs are reduced 10%?

Example 1.3 (solution to a)

\[ Q_{BE} = \frac{CF}{p - v} = \frac{2,024,000}{85.56 \text{ / hour} - 62 \text{ / hour}} = 85,908 \text{ hours} \]
Example 1.3 (solution to a)

\[ Q_{BE} \% = \frac{Q_{BE}}{\text{total CAPACITY}} = \frac{85,908 \text{ hours}}{160,000 \text{ hours}} = 0.54 = 54\% \text{ of total CAPACITY} \]

Example 1.3 (solution to a)

If \( CF \) is reduced by 10%,

\[ \text{new}Q_{BE} = ? \quad \text{and} \quad (Q_{BE} - \text{new}Q_{BE}) / Q_{BE} = ? \]

Reduced \( CF = CF \times (1 - 0.1) = \$2,024,000 \times 0.9 = \$1,821,600 \)

\[ \text{new}Q_{BE} = \frac{\text{reduced } CF}{p - v} = \frac{\$1,821,600}{\$85.56 \text{ / hour} - \$62 \text{ / hour}} = 77,318 \text{ hours} \]

\[ \% \text{ reduction in } Q_{BE} = \frac{Q_{BE} - \text{new}Q_{BE}}{Q_{BE}} = \frac{85,908 \text{ hrs} - 77,318 \text{ hrs}}{85,908 \text{ hrs}} = 10\% \]
The relationship between price and demand can be expressed as a linear function:

\[ Q = \frac{a}{b} - p \left( \frac{1}{b} \right) = \frac{(a - p)}{b} \]

or

\[ p = a - bQ \quad \text{for} \ 0 \leq Q \leq \frac{a}{b}, \ \text{and} \ a > 0, \ b > 0 \]

where
- \( p \) = price per unit
- \( Q \) = demand for the product or service (# of units)
- \( a \) = base (maximum) price (intercept on the price axis)
- \( b \) = slope of the price-Demand line
- \( 1/b \) = amount by which demand increases for each unit decrease in price
The Total Revenue Function

\[ TR = (\text{price per unit}) \times (\text{demand}) = pQ \]

\[ TR = (a - bQ)Q = aQ - bQ^2 \text{ for } 0 \leq Q \leq a/b \]

where
- \( p \) = price per unit
- \( Q \) = demand for the product or service (# of units)
- \( a \) = base (maximum) price (intercept on the price axis)
- \( b \) = slope of the price-Demand line

The Total Revenue Function

\[ TR = aQ - bQ^2 \]

Maximum Revenue
\[ \max TR = a^2/4b \]
at \( Q^* = a/2b \)
Nonlinear Breakeven Analysis

Breakeven Points

\[ TR = CT \Rightarrow TR - CT = 0 \]

\[ TR = aQ - bQ^2 \text{ and } CT = CF + vQ \]

\[-bQ^2 + (a - v)Q - CF = 0\]

\[ Q_{BE-1,2} = \frac{-(a - v) \pm \sqrt{(a - v)^2 - 4(-b)(-CF)}}{2(-b)} \]

Profitable Domain: \( Q_{BE-1} < Q_{demand} < Q_{BE-2} \)
Profit Function

\[ \text{PROFIT} = TR - CT = -bQ^2 + (a - v)Q - CF \]

for \( 0 \leq Q \leq a/b \), and \( a > 0, b > 0 \)

![Graph showing profit function]

Optimal Production Quantity

\[ \text{PROFIT} = TR - CT = -bQ^2 + (a - v)Q - CF \]

\[ \frac{d(\text{PROFIT})}{dQ} = 0 \quad \Rightarrow \quad -2bQ + (a - v) = 0 \]

\[ Q^* = \frac{(a - v)}{2b} \]

\[ \frac{d^2(\text{PROFIT})}{dQdQ} < 0 \quad \Rightarrow \quad -2b < 0, \quad \text{for } b > 0 \]

It proves that \( Q^* \) maximizes profit
Example 1.6

A company produces an electronic timing switch that is used in consumer and commercial products made by several other manufacturing firms. The fixed cost \((CF)\) is $73,000 per month, and the variable cost \((v)\) is $83 per unit. The selling price per unit is \(p = 180 - 0.02Q\). For this situation;

a) Find the volumes at which breakeven occurs; that is, what is the domain of profitable demand?
b) Determine the optimal volume for this product in order to maximize profit.
c) What is the maximum profit per month at the optimal volume?

Example 1.6 (solution to a)

\[
TR = CT \quad \text{or Profit} = 0
\]

\[
Profit = TR - CT = pQ - CF - vQ = 0
\]

\[
Profit = (180 - 0.02Q)Q - 73,000 - 83Q = 0
\]

\[
-0.02Q^2 + (180 - 83)Q - 73,000 = 0
\]

\[
Q_{BE-1} = \frac{-97 + \sqrt{(97)^2 - 4(-0.02)(-73,000)}}{2(-0.02)} = 932 \text{ units / month}
\]

\[
Q_{BE-2} = \frac{-97 - \sqrt{(97)^2 - 4(-0.02)(-73,000)}}{2(-0.02)} = 3,918 \text{ units / month}
\]

The domain of profitable demand per month = 932 units to 3,918 units
Example 1.6 (solution to b)

\[ \text{Profit} = (180 - 0.02Q)Q - 73,000 - 83Q \]
\[ \text{Profit} = -0.02Q^2 + (180 - 83)Q - 73,000 \]

Take first derivative and set \( = 0 \)

\[ \frac{d\text{Profit}}{dQ} = 0 \Rightarrow -0.04Q + 97 = 0 \]

\( Q^* = 2,425 \text{ units /month} \) (# of products that maximizes profit)

\[ \frac{d^2\text{Profit}}{dQ^2} = -0.04 < 0 \]

It proves that \( Q^* \) maximizes profit

Example 1.6 (solution to c)

Use equation for profit and \( Q^* = 2,425 \) units /month, \n\[ \text{Profit} = (180 - 0.02Q)Q - 73,000 - 83Q \]
\[ \text{Profit} = -0.02Q^2 + (180 - 83)Q - 73,000 \]

Maximum \( \text{Profit} = -0.02(2,425)^2 + (97)(2,425) - 73,000 \)
Maximum \( \text{Profit} = $44,612 /\text{month} \)
Life Cycle Cost

- Summation of all the costs, both recurring and nonrecurring, related to a product, structure, system, or service during its life span.

- **LCC** = Investment Costs
  + O&M Costs
  + Replacement Costs
  + Energy Costs
  + Disposal Costs
  - Salvage Value (if any)
Cost Driven Design Optimization

1. Identify the design variable that is the primary cost driver.
2. Express the cost model in terms of the design variable.
3. For continuous cost functions, differentiate to find the optimal value. For discrete functions, calculate cost over a range of values of the design variable.
4. Solve the equation in step 3 for a continuous function. For discrete, the optimum value has the minimum cost value found in step 3.

A simplified cost function.

\[ \text{Cost} = aX + \frac{b}{X} + k \]

where,

- \( a \) is a parameter that represents the directly varying cost(s),
- \( b \) is a parameter that represents the indirectly varying cost(s),
- \( k \) is a parameter that represents the fixed cost(s), and
- \( X \) represents the design variable in question.
Example 2.6 on pg.59

How fast should the airplane fly?

\[ C_o = \text{operating cost} = k \cdot n \cdot v^{3/2} \]

where,
\[ k = \text{a constant of proportionality} \]
\[ n = \text{the trip length in miles} \]
\[ v = \text{velocity in mile per hour} \]

\[ C_c = \text{time cost} = 300,000 \left( \frac{n}{v} \right) \]

Solution

\[ C_T = C_o + C_c = k \cdot n \cdot v^{3/2} + 300,000 \left( \frac{n}{v} \right) \]

\[ \frac{dC_T}{dv} = 0 \rightarrow \frac{3}{2} k \cdot n \cdot v^{1/2} - 300,000 \left( \frac{n}{v^2} \right) = 0 \]

\[ k = 0.0375 \rightarrow 0.05625 \cdot v^{5/2} - 300,000 = 0 \]

\[ v^* = \left( \frac{300,000}{0.05625} \right)^{0.4} = 490.68 \text{ mph.} \]
Present Economy Studies

- Alternatives are being compared over one year or less.
- When revenues and other economic benefits vary among alternatives, choose the alternative that maximizes overall profitability of defect-free output.
- When revenues and other economic benefits are not present or are constant among alternatives, choose the alternative that minimizes total cost per defect-free unit.

Example 2.9 on pg.64

<table>
<thead>
<tr>
<th></th>
<th>Machine A</th>
<th>Machine B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production rate</td>
<td>100 parts/hour</td>
<td>130 parts/hour</td>
</tr>
<tr>
<td>Available hours</td>
<td>7 hours/day</td>
<td>6 hours/day</td>
</tr>
<tr>
<td>Defective ratio</td>
<td>3%</td>
<td>10%</td>
</tr>
</tbody>
</table>

Material cost = $6 per part    Selling price = $12 per part
Operator cost = $15 per hour  Overhead cost = $5 per hour

(a) Which machine should be selected?
(b) What is the breakeven defective ratio?
Solution to (a)

**Machine A:**
Profit = (100)(7)(12)(1 – 0.03) – (100)(7)(6) – (7)(20)
= $3,808 per day

**Machine B:**
Profit = (130)(6)(12)(1 – 0.10) – (130)(6)(6) – (6)(20)
= $3,624 per day

Therefore, select **machine A** to maximize profit per day

Solution to (b)

X, breakeven defective ratio for machine B


X = 0.08