Objective:

- To evaluate the economic profitability and liquidity of a single proposed investment project.

Equivalent measures of a project’s profitability

- Present Worth (PW)
- Future Worth (FW)
- Annual Worth (AW)
- Internal Rate of Return (IRR)
- External Rate of Return (ERR)
CHAPTER 4

Measures of liquidity

- Simple Payback Method (Ø)
- Discounted Payback Method (Ø')

CHAPTER 4

Example Problem for Project Evaluation Methods

Cost/Revenue Estimates

- Initial Investment: $50,000
- Annual Revenues: 20,000
- Annual Operating Costs: 2,500
- Salvage Value @ EOY 5: 10,000
- Study Period: 5 years
- MARR: 20% per year

CHAPTER 4

Cash flow diagram
Present Worth (PW) Method

- Compute the present equivalent of the estimated cash flows using the MARR as the interest rate.
- If PW (MARR) \( \geq 0 \), then the project is profitable.
- If PW (MARR) < 0, then the project is not profitable.

PW Method

- \( PW(20\%) = -50,000 + (20,000 - 2,500)(P/A, 20\%, 5) + 10,000(P/F, 20\%, 5) = $6,354.50 \)
- Since \( PW(20\%) \geq 0 \), the project is profitable.

PW = $6,354.50 tells us:

- We have recovered our entire $50,000 investment,
- We have earned our desired 20% on this investment,
- We have made a lump sum equivalent profit of $6,354.50 beyond what was expected (required).
FW Method

- FW(20%) = – 50,000(F/P,20%,5) + (20,000-2,500)(F/A,20%,5) + 10,000
  = $15,813
- Since FW(20%) ≥ 0, the project is profitable.

Annual Worth (AW) Method

- AW(i%) = R – E – CR(i%)
where
  - R = annual equivalent revenues
  - E = annual equivalent expenses
  - CR = annual equivalent capital recovery cost

AW Method

- CR(i%) = I(A/P,i%,N) – S(A/F,i%,N)
  = $50,000(A/P,20%,5) – $10,000(A/F,20%,5)
  = $16,720 – $1,344 = $15,376
- AW(20%) = R – E – CR(20%)
  = $20,000 - $2,500 - $15,376
  = $2,124
- Since AW(20%) ≥ 0, project is profitable
Equivalent Worth Methods

- If \( PW \geq 0 \), then \( FW \geq 0 \) and \( AW \geq 0 \).
- From our example,
  - \( PW = $6,354.50 \) therefore,
  - \( FW = 6,354.50(F/P, 20\%, 5) = $15,812 \) and
  - \( AW = 6,354.50(A/P, 20\%, 5) = $2125 \)

IRR (Internal Rate of Return)

- IRR is the interest rate that makes the PW, AW, and FW of a project's estimated cash flows equal to zero.
- That is, \( PW(i') \) of cash inflow = \( PW(i') \) of cash outflow.

IRR (Internal Rate of Return)

- We commonly denote the IRR by \( i' \).
  - \( PW(i' \%) = 0 \)
  - \( AW(i' \%) = 0 \)
  - \( FW(i' \%) = 0 \)
- In general, we must solve for \( i' \) by trial and error.
Evaluating Projects with the IRR

- Once we know the value of the IRR for a project, we compare it to the MARR to determine whether or not the project is acceptable with respect to profitability.
- \[ IRR = i' \geq MARR \] project is acceptable
- \[ IRR = i' < MARR \] project is unacceptable (reject)

IRR method

- Find \( i' \) such that the PW(\( i' \)) = 0.
- \[ 0 = -50,000 + 17,500(\text{P/A, } i'\%, 5) + 10,000(\text{P/F, } i'\%, 5) \]
- PW (20\%) = 6354.50 tells us that \( i' > 20\% \)
- PW (25\%) = 339.75 > 0, tells us that \( i' > 25\% \)
- PW (30\%) = -4,684.24 < 0, tells us that \( i' < 30\% \)
- 25\% < \( i' < 30\% \)
- Use linear interpolation to estimate \( i' \).

IRR - Linear Interpolation

<table>
<thead>
<tr>
<th>( i' )</th>
<th>PW</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) 25</td>
<td>339.75</td>
</tr>
<tr>
<td>(b) ( i' )</td>
<td>0</td>
</tr>
<tr>
<td>(c) 30</td>
<td>-4684.24</td>
</tr>
</tbody>
</table>
IRR - Linear Interpolation

\[ i' = 25 + \frac{339.75 - 0}{339.75 - (-4684.24)} \times (30 - 25) = 25.3\% > MARR \]

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ERR (External Rate of Return)

- In other words, the ERR is the interest rate that makes the PW (MARR%) of costs equivalent to the FW(MARR%) of revenues.
- PW (ε%) of cash outflow (F/P, i', N) = FW(ε%) of cash inflow.
- ε% = MARR% (unless recovered funds are reinvested at a different interest rate rather than MARR).

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ERR Method

\[ (F / P, i'\%, N) = (1 + i')^N \]
ERR Method

\[ \sum_{k=0}^{N} E_k(P/F, \epsilon\%, k) = \sum_{k=0}^{N} R_k(F/P, \epsilon\%, N-k) \]

\[ (1+i')^N = \frac{\sum_{k=0}^{N} R_k(F/P, \epsilon\%, N-k)}{\sum_{k=0}^{N} E_k(P/F, \epsilon\%, k)} \]

\[ i' = \sqrt{\frac{\sum_{k=0}^{N} R_k(F/P, \epsilon\%, N-k)}{\sum_{k=0}^{N} E_k(P/F, \epsilon\%, k)} - 1} \]
Evaluating Projects with the ERR

- Once we know the value of the ERR for a project, we compare it to the MARR (not to ε) to determine whether or not the project is acceptable with respect to profitability.
- ERR = \( i' \geq \) MARR project is acceptable
- ERR = \( i' < \) MARR project is unacceptable (reject)

ERR method

- Find \( i\% \) such that 
  \[ \text{PW}(\% \text{ of costs})(F/P, \% , N) = \text{FW}(\% \text{ of revenues}) \]
- \( \text{FW}(20\%) \text{ [revenues]} = (20,000 - 2,500)(F/A,20\%,5) + 10,000 = $140,228 \)
- \( \text{PW}(20\%) \text{ [costs]} = $50,000 \)
- \( $50,000(1 + i')^N = $140,228 \)

ERR method

\[
\frac{\$50,000(i + i')^N}{\$50,000} = \frac{\$140,228}{\$50,000} = 2.8045
\]

\[ i' = 22.9\% > \text{MARR}, \text{ therefore project is acceptable} \]
Measures of Liquidity

- **Simple Payback Period (Ø)** – how many years it takes to recover the investment (ignoring the time value of money).
- **Discounted Payback Period (Ø’)** – how many years it takes to recover the investment (including the time value of money).

### Simple Payback Period

<table>
<thead>
<tr>
<th>EOY</th>
<th>Simple Payback Cumulative PW (i = 0%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>- $50,000</td>
</tr>
<tr>
<td>1</td>
<td>- 32,500 = - $50,000 + 17,500</td>
</tr>
<tr>
<td>2</td>
<td>- 15,000 = - 32,500 + 17,500</td>
</tr>
<tr>
<td>3</td>
<td>+ 2,500 = - 15,000 + 17,500</td>
</tr>
<tr>
<td>4</td>
<td>+ 20,000 = 2,500 + 17,500</td>
</tr>
<tr>
<td>5</td>
<td>+ 47,500 = 2,500 + 17,500 + 10,000</td>
</tr>
</tbody>
</table>

Payback Period Ø = 3 years

### Discounted Payback Period

<table>
<thead>
<tr>
<th>EOY</th>
<th>Discounted Payback Cumulative PW (i = MARR = 20%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>- $50,000</td>
</tr>
<tr>
<td>1</td>
<td>- 35,417 = - $50,000 + 17,500(P/F, 20%, 1)</td>
</tr>
<tr>
<td>2</td>
<td>- 23,264 = - 35,417 + 17,500(P/F, 20%, 2)</td>
</tr>
<tr>
<td>3</td>
<td>- 13,137 = - 23,264 + 17,500(P/F, 20%, 3)</td>
</tr>
<tr>
<td>4</td>
<td>- 4,697 = - 13,137 + 17,500(P/F, 20%, 4)</td>
</tr>
<tr>
<td>5</td>
<td>+ 6,354.50 = - 4,697 + 27,500(P/F, 20%, 5)</td>
</tr>
</tbody>
</table>

Discounted Payback Period Ø’ = 5 years
BENEFIT / COST RATIO

- The benefit/cost ratio method involves the calculation of a ratio of benefits to costs.
- The B/C ratio is defined as the ratio of the equivalent worth of benefits to the equivalent worth of costs.
- Generally, PW or AW is used as equivalent worth measure in B/C ratio.

B/C using PW method:

\[
\frac{B}{C} = \frac{PW(B)}{I + PW(O & M)}
\]

- \(PW(x)\) = present worth of \(x\)
- \(B\) = benefits of the proposed project
- \(I\) = initial investment in the proposed project
- \(O&M\) = operating and maintenance costs of the proposed project.

B/C with salvage value

\[
\frac{B}{C} = \frac{PW(B)}{[I + PW(O & M)] - PW(S)}
\]

- \(S\) = Salvage value (market value)
EVALUATING PROJECT WITH B/C RATIO METHOD

- Once we compute the B/C ratios for all above formulations using the MARR as the interest rate.
- If $B/C \geq 1.0$, then the project is profitable (acceptable)
- If $B/C < 1.0$, then the project is not profitable (rejected).

B / C RATIO EXAMPLE

Estimated cash flows of the investment:

- Construction cost = $850,000
- Purchasing cost of land = $350,000
- Investment cost = $1,200,000
- Annual maintenance cost = $26,500
- Annual operating cost = $75,000
- Monthly salary = $8,000×12
- Annual expenses = $197,500
- Annual revenues = $490,000

B / C RATIO EXAMPLE

- Apply the B/C ratio method with a study period of 20 years for MARR of 10% to determine whether the project will be implemented or not without considering the salvage value.
B/C solution using PW

- PW(cost)@10% = 1,200,000
  + 197,500(P/A, 10%, 20)
  = $2,881,436
- PW(benefit)@10% = 490,000(P/A, 10%, 20)
  = $4,171,664
- B/C = (4,171,664 / 2,881,436) = 1.45 > 1.0
the project is profitable.