Objective

- Analyze the economic consequences of engineering projects where uncertainty exists, without making probability of risk explicit
Tools for dealing with uncertainty

- Break-even analysis
- Sensitivity graph
- Risk – adjusted MARR
- Using discrete random variables
- Using continuous random variables

Example

An investment of $1,000,000 is needed to found a plant producing hydraulic pumps. The details of operating and maintenance costs are as follows: the fixed cost is $50,000 per month, and the variable cost is $155 per pump. Sales price is $332 per pump. What is the breakeven point in number of pumps per month? When the study period is 5 years and MARR=20%
Solution:

\[ 12(177Q - 50,000)(P/A, 20\%, 5) = 1,000,000 \]

Monthly net revenue

Annual net revenue

Present equivalent net revenue for 5 years

\[ Q = 440 \text{ pumps per month} \]

Case problem

- Suppose that there are two alternative electric motors that provide 100 hp output.
- An Alpha motor can be purchased for $12,500 and has an efficiency of 74%, an estimated life of 10 years, and estimated maintenance cost of $500 per year.
- A Beta motor will cost $16,000 and has an efficiency of 92%, a life of 10 years, and annual maintenance costs of $250.
- Annual taxes and insurance costs on either motor will be 1.5% of the investment.
- If the minimum attractive rate of return is 15%, how many hours per year would the motors have to be operated at full load for the annual costs to be equal?
- Assume that salvage values for both motors are negligible and that electricity costs $0.05 per kilowatt-hour.
Setup the equations

- Decision Criterion: Minimize Equivalent Uniform Annual Cost (EUAC)
- EUAC(15%) = CR(15%) + Operating cost + Taxes & Ins. + Maintenance

\[ \text{Electrical Efficiency} = \frac{\text{output power}}{\text{input power}} \]

\[ 1 \text{ hp} = 0.746 \text{ kW} \]

Solving for Q

\[ \text{CR}(15\%)_\alpha = 12,500 \ (A/P, 15\%, 10) = \$2,490 \]
\[ \text{CR}(15\%)_\beta = 16,000 \ (A/P, 15\%, 10) = \$3,190 \]

At breakeven, EUAC_\alpha = EUAC_\beta

\[ 2,490 + 5.04Q + 500 + 187 = 3,190 + 4.05Q + 250 + 240 \]
\[ 3,177 + 5.04Q = 3,680 + 4.05Q \]
\[ Q = 508 \text{ hours/year} \]
INTERPRET THE SOLUTION

SENSITIVITY ANALYSIS

- Suppose for a project the following most likely estimates are given as:
  - Investment cost: $95,000
  - Net annual receipts: $20,000
  - Salvage value: $10,000
  - MARR: 12% per year
  - Study Period: 10 years
- Investigate the PW at −50% and +50% changes in factors *investment cost, net annual receipts, and salvage value*. Determine the decision reversal points.
Setup the PW for most likely estimates

\[
PW(12\%) = -95,000 + 20,000(P/A, 12\%, 10) + 10,000(P/F, 12\%, 10) = $21,224
\]

– 50% in Investment cost

\[
I = 95,000 \times (1 - 0.5) = $47,500
\]

\[
PW(12\%) = -47,500 + 20,000(P/A, 12\%, 10) + $10,000(P/F, 12\%, 10) = $68,724
\]
+ 50% in Investment cost

\[ I = 95,000 \times (1 + 0.5) = $142,500 \]

\[
PW(12\%) = -142,500 + 20,000 \times (P/A, 12\%, 10) + 10,000 \times (P/F, 12\%, 10) = -$26,276
\]

Decision reversal point of Investment cost

\[
PW \ (12\%) = 0 = -I + 20,000(P/A, 12\%, 10) + 10,000(P/F, 12\%, 10)
\]

\[ I = $116,224 \]

*investment cost can increase to this value without making the project unattractive*

Increase\% = \[
\frac{(116,224 - 95,000)}{95,000} \times 100 = 22.3\%
\]
Results of sensitivity analysis

<table>
<thead>
<tr>
<th></th>
<th>PW at (-/+ 5%)</th>
<th>Decision Reversal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>change in factor value</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- 50%</td>
<td>+ 50%</td>
</tr>
<tr>
<td>Investment</td>
<td>$68,724</td>
<td>-$26,276</td>
</tr>
<tr>
<td>Net revenue</td>
<td>-$35,278</td>
<td>$77,726</td>
</tr>
<tr>
<td>Salvage value</td>
<td>$19,614</td>
<td>$22,834</td>
</tr>
</tbody>
</table>

PROBABILISTIC APPROACH
EXAMPLE 1

- A project under consideration is thought to involve a medium degree of risk. The risk–free discount rate is 4% and the appropriate risk premium is believed to be 6%. Capital investment = $1000 (it is known with certainty)
The project has the following estimated cash flows:

<table>
<thead>
<tr>
<th>Market Conditions</th>
<th>End – of – Year</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good</td>
<td>1000 1000 1000 1000 1000 1000</td>
<td>0.10</td>
</tr>
<tr>
<td>Med./good</td>
<td>800 800 800 800 800 800</td>
<td>0.20</td>
</tr>
<tr>
<td>Medium</td>
<td>600 600 600 600 600 600</td>
<td>0.40</td>
</tr>
<tr>
<td>Med./poor</td>
<td>400 400 400 400 400 400</td>
<td>0.20</td>
</tr>
<tr>
<td>Poor</td>
<td>200 200 200 200 200 200</td>
<td>0.10</td>
</tr>
</tbody>
</table>

a) Determine the expected value and standard deviation of the project’s PW

Expected value of PW

MARR = 4% + 6% = 10% risk-adjusted MARR

\[(A) \times (B) = (A) \times (B)\]

<table>
<thead>
<tr>
<th>Market Condition</th>
<th>Probability</th>
<th>PW (10%)</th>
<th>Expected Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good</td>
<td>0.10</td>
<td>$2,791</td>
<td>279</td>
</tr>
<tr>
<td>M/good</td>
<td>0.20</td>
<td>2,033</td>
<td>407</td>
</tr>
<tr>
<td>Medium</td>
<td>0.40</td>
<td>1,274</td>
<td>510</td>
</tr>
<tr>
<td>M/poor</td>
<td>0.20</td>
<td>516</td>
<td>103</td>
</tr>
<tr>
<td>Poor</td>
<td>0.10</td>
<td>-242</td>
<td>-24</td>
</tr>
</tbody>
</table>

\[PW = -1000 + 1000(P/A,10\%,6) = 2791\]
\[PW = -1000 + 800(P/A,10\%,6) = 2033\]
\[PW = -1000 + 600(P/A,10\%,6) = 1274\]
\[PW = -1000 + 400(P/A,10\%,6) = 516\]
\[PW = -1000 + 200(P/A,10\%,6) = -242\]

\[E[PW] = $1,274\]
### Standard deviation of PW

**Table:**

<table>
<thead>
<tr>
<th>Market Condition</th>
<th>PW (10%)</th>
<th>Probability</th>
<th>(PW – E[PW])²</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good</td>
<td>$2,791</td>
<td>0.10</td>
<td>$2,300,642</td>
<td>230064</td>
</tr>
<tr>
<td>M/good</td>
<td>2,033</td>
<td>0.20</td>
<td>575519</td>
<td>115104</td>
</tr>
<tr>
<td>Medium</td>
<td>1,274</td>
<td>0.40</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>M/poor</td>
<td>516</td>
<td>0.20</td>
<td>574087</td>
<td>114817</td>
</tr>
<tr>
<td>Poor</td>
<td>-242</td>
<td>0.10</td>
<td>2297779</td>
<td>2297778</td>
</tr>
</tbody>
</table>

\[(2791 – 1274)^2 = 2,300,642\]

\[\text{Var}[PW] = 689763\]

\[\text{Std.Dev.} = \sqrt{\text{Var}[PW]} = 831\]

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### EXAMPLE 2

- A firm is considering purchasing a new 20-hp electric motor
- Motor A sells for $800 and has an efficiency rating of 90%
- Motor B sells for $600 and a rating of 84%.
- The cost of electricity is $0.038/kWh.
- An 8-yr study period is used; equal salvage values are assumed for the two motors. A MARR of 20% is to be used.
- Note: 1 hp = 0.75 kW
EXAMPLE 2 cont’d

- The annual usage of motors is uncertain, and the following probabilities have been estimated:

<table>
<thead>
<tr>
<th>Annual usage</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000 hours</td>
<td>0.2</td>
</tr>
<tr>
<td>2000</td>
<td>0.4</td>
</tr>
<tr>
<td>3000</td>
<td>0.3</td>
</tr>
<tr>
<td>4000</td>
<td>0.1</td>
</tr>
</tbody>
</table>

- Which motor do you prefer?

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Operating cost of electric motors

\[ E_A = 0.9 = \frac{20 \text{ hp}}{\text{input}_A} \quad \text{E}_B = 0.84 = \frac{20 \text{ hp}}{\text{input}_B} \]

\[ \text{input}_A = 22.22 \text{ hp} \quad \text{input}_B = 23.8 \text{ hp} \]

\[ 22.22 \times 0.75 = 16.67 \text{ kW} \quad 23.8 \times 0.75 = 17.86 \text{ kW} \]

\[ \text{Opr.cost of A} = 16.67 \text{ kW/hr} \times $0.038 /\text{kW} \times Q \text{ hrs} \]
\[ = $0.63 / \text{hr} \times Q \text{ hrs} \]

\[ \text{Opr.cost of B} = 17.86 \text{ kW/hr} \times $0.038 /\text{kW} \times Q \text{ hrs} \]
\[ = $0.68 / \text{hr} \times Q \text{ hrs} \]

Q? = Expected annual usage of motors
Q (expected annual usage)

<table>
<thead>
<tr>
<th>Annual usage</th>
<th>Probability</th>
<th>Expected value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000 hours</td>
<td>0.2</td>
<td>200</td>
</tr>
<tr>
<td>2000</td>
<td>0.4</td>
<td>800</td>
</tr>
<tr>
<td>3000</td>
<td>0.3</td>
<td>900</td>
</tr>
<tr>
<td>4000</td>
<td>0.1</td>
<td>400</td>
</tr>
</tbody>
</table>

Q = 2300 hours

AW(20%) of electric motors

\[ AW(20\%) = CR(20\%) + \text{Operating cost} \]

\[ AW(20\%)_A = 800(A/P,20\%,8)+0.63(2300) = 208.49 + 1449 = $1657.49 \]

\[ AW(20\%)_B = 600(A/P,20\%,8)+0.68(2300) = 156.37 + 1564 = $1720.37 \]

Electric motor of A is selected!
Example 3

Suppose that annual net revenue of a project is random variable and distributed normally with a mean value of $4000 and a standard deviation of $1200. This project requires $15000 investment and has no market value at the end of its useful life of 7 years. If company desires 8% annual return on investment, find the probability of loss on this project.

Solution

\[
\text{PW(8\%)} = 0 = -15,000 + A(P/A, 8\%, 7)
\]

\[A = 2,881\]

\[z = \frac{x - \mu}{\sigma} = \frac{2,881 - 4,000}{1,200} = -0.93\]

\[F(z) = \text{Cumulative probability of given } z\]

\[F(-0.93) = 0.1762 = 17.62\%\]

Probability of loss on this project is 17.62\%