Inventory Management

- Inventory is a “buffer” between supply and demand

\[ S(t) \rightarrow \text{Inventory} \rightarrow D(t) \]

Supply process (+)  Demand process (–)

- \( S(t) \) and \( D(t) \) have different rates and timings due to endogenous and exogenous factors.
Factors for Inventory

- Endogenous factors: are a matter of policy
  - Economies of scale
  - Operation smoothing
  - Customer service
- Exogenous factors: are uncontrollable
  - Uncertainty

Inventory Types

Types are classified according to the value added during the manufacturing process:
- Raw material inventories
- Work-in-process (WIP) inventories
- Finished goods inventories
An inventory system is a coordinated set of rules and procedures that allow for routine decisions on “when” and “how much” to order of each item needed in the manufacturing / procurement process to fill customer demand.

Objective of the system is to minimize the costs incurred in the inventory system, attaining at the same time customer service level specified by the company policy.
Costs in an Inventory System

Purchasing / production cost  Inventory holding cost  Ordering / setup cost  Stockout cost

Total Inventory Cost

Purchasing / production cost

- Sometimes these unit costs are independent of the quantity purchased / produced
- But sometimes there may be “quantity discounts”
  - As quantity increases, unit cost decreases
Inventory Holding Cost

Components

<table>
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<th>Components</th>
<th>%</th>
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<td>1- Opportunity cost</td>
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<tr>
<td>2- Insurance cost</td>
<td>2 – 4</td>
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<td>3- Taxes</td>
<td>1 – 3</td>
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<tr>
<td>4- Storing cost</td>
<td>1 – 3</td>
</tr>
<tr>
<td>5- Damage, spoilage, obsolescence</td>
<td>4 – 10</td>
</tr>
<tr>
<td>Total inventory holding cost ⇒</td>
<td>20 – 40 % = i</td>
</tr>
</tbody>
</table>

\[ H = i \cdot c \]

Cost to carry one unit of inventory for one unit of time

Cost of carrying $1 of inventory for one unit of time

Cost / unit
Ordering / setup cost

- Generally independent of quantity ordered / produced
- **Ordering cost**: cost of preparing and monitoring the order
- **Setup cost**: cost of preparing the machine for the production run

Stockout / shortage cost

- Occurs when there is a demand for an out-of-stock item
- **Shortage**:  
  - Backordering: delay in cash flow, loss of goodwill  
  - Lost sales: lost profit, loss of goodwill
EOQ Models

- Deterministic Single-Item Models with Static Demand
- Rate of demand for the commodity is known with certainty and constant
- Lead time and other system parameters are known with certainty, constant and independent of the quantity ordered
- All shortages are backlogged

Production rate and stockout assumptions for the EOQ Models

<table>
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<tr>
<th>Model</th>
<th>Production rate</th>
<th>Shortage cost</th>
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<td>Infinite</td>
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<td>II (simple EPQ)</td>
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<td>IV (EPQ with backlogging)</td>
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<td>Finite</td>
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</table>
Model I (Simple EOQ)

- Simplest inventory model
- Known as “classical economic lot-size model” or “Wilson lot-size formula”
- Suitable for raw material purchase in production or for a retail environment

Notation

D = demand rate (units / year)
A = fixed ordering cost ($ / order)
H = annual inventory holding cost per unit
   ($ / unit / year)
c = unit variable cost of procurement
i = annual inventory carrying cost rate
H = i . c
TC = total average annual cost of inventory
Q = order quantity
Fixed reorder quantity system

Average inventory

\[ \text{Average inventory per time-unit} = \frac{Q \cdot T}{2} = \frac{Q}{2} \]
Inventory Holding Cost

Increase in lot size results in an increase in Inventory Holding Cost

Ordering cost

**Components**

- Purchasing cost per order $= cQ$
  (purchasing cost of order size $Q$)

- Fixed cost per order $= A$
  (fixed cost per order including paper work, computer processing, telephone calls, postage, transportation, handling, inspection.)
Ordering cost

Total cost per order = $cQ + A$

Number of orders per year = $D / Q$,
where, $D =$ Annual demand, $Q =$ lot size per order

Annual cost of order = $(cQ + A) (D/Q)$
= $cD + A(D/Q)$,
$cD =$ annual purchasing cost $A(D/Q) =$ annual ordering cost.
Total Inventory Cost

$$TC(Q) = cD + \frac{AD}{Q} + H\frac{Q}{2}$$

Where,
- $c$ = unit cost ($/unit$)
- $D$ = annual demand (units)
- $A$ = fixed ordering cost ($/order$)
- $H$ = annual inventory holding cost ($/unit/year$)
- $Q$ = lot size (quantity to be purchased per order)

Economic Order Quantity (EOQ)

$$TC(Q) = A\frac{D}{Q} + H\frac{Q}{2}$$

Minimum total cost

Setup cost = Holding cost

$$Q^* \text{ (EOQ)}$$

Cost ($$$/$$)$
Economic Order Quantity (EOQ)

\[
\frac{dTC(Q)}{dQ} = 0 \implies -\frac{AD}{Q^2} + \frac{H}{2} = 0
\]

\[Q^* = \sqrt{\frac{2AD}{H}}\]

\(Q^*\) = Economic Order Quantity, EOQ (optimal lot size)

\[\frac{AD}{Q} = \frac{Qic}{2} \implies Q^2 = \frac{2AD}{ic} \implies Q^* = \sqrt{\frac{2AD}{ic}}\]

EOQ-Example

Two weeks later

- Lead time = 2 weeks
- \(Q\) (order quantity)?
- \(A = $1000\) (fixed cost per order)
- \(c = $500\) per unit
- \(i = 20\%\)
- Consumption rate = 4 parts/hr
- Available time = 40 hours/wk
EOQ-Example Continued

- Suppose plant should work in its full capacity for 52 weeks in one year.
- Annual consumption of raw material in plant = \( D \) = annual demand from supplier

\[
D = 160 \times 52 = 8320 \text{ parts/year}
\]

Economic Order Quantity (EOQ)

\[
Q^* = \sqrt{\frac{2AD}{H}} \quad \Rightarrow \quad EOQ = \sqrt{\frac{2(1000)(8320)}{100}} = 408 \text{ units}
\]

Each inventory period length = \( Q^* / D \) = 408 / 8320 = 0.049 years

0.049 \times 52 \text{ weeks} = 2.55 \text{ weeks}

2.55 \times 40 \text{ hours} = 102 \text{ hours}
Model II (Simple EPQ)

- It is the case in which the order is being filled by a machine, which has an infinite production rate

- **Additional symbols:**
  - \( P \) = Production (supply) rate (units/year)
  - \( S \) = Setup cost (\$/lot)
Additional Assumption

i. $P > D$; that is, the supply rate exceeds the demand rate

ii. The supplying process is continuous and takes place at a constant rate during the period

iii. Setup cost is independent of quantity

Model II (Simple EPQ)

Finite input rate, no backlogging
Model II (Simple EPQ)

\( tp = \) production period
\( td = \) depletion period
\( P - D = \) inventory accumulation rate
\( D = \) inventory depletion rate
\( T = \) inventory cycle length = \( tp + td = Q / D \)
\( I_{\text{max}} = \) maximum inventory = \( (P - D)tp \)

Average inventory = \( I_{\text{max}} (T) / 2 / T = I_{\text{max}} / 2 \)

\( I_{\text{max}} = (P - D) tp \)
\( tp(P) = Q \)

\( I_{\text{max}} = (P - D) \frac{Q}{P} = Q \left( 1 - \frac{D}{P} \right) \)

Average inventory = \( \frac{Q}{2} \left( 1 - \frac{D}{P} \right) \)
Inventory Holding Cost

Inventory holding cost in continuous replenishment will have less value than that in instantaneous replenishment

Economic Production Quantity

\[ TC(Q) = \frac{D}{Q} + \frac{Q}{2} \left( \frac{1}{P} \right) ic \]

Holding Cost = \( \frac{Q}{2} ic \)

Setup cost = \( \frac{D}{Q} \)

Ordering cost = \( \frac{D}{Q} \)
Economic Production Quantity

\[ \frac{dTC(Q)}{dQ} = 0 \quad \Rightarrow \quad \frac{-SD}{Q^2} + \frac{H}{2} \left(1 - \frac{D}{P}\right) = 0 \]

\[ Q^* = \sqrt{\frac{2SDP}{H(P - D)}} \quad \Rightarrow \quad Q^* = \sqrt{\frac{2SD}{H \left(1 - \frac{D}{P}\right)}} \]

Q* = Economic Production Quantity, EPQ (optimal batch size)

Economic Production Quantity

\[ \frac{SD}{Q} = \frac{Qic}{2} \left(1 - \frac{D}{P}\right) \quad \Rightarrow \quad Q^2 = \frac{2SD}{ic \left(1 - \frac{D}{P}\right)} \]

\[ \Rightarrow \quad Q^* = \sqrt{\frac{2SD}{ic \left(1 - \frac{D}{P}\right)}} \]
EPQ- Example

Suppose for a certain product type you need to produce weekly uniform demand as stated below:

- Production rate = 4 parts/hr
- Available hours per week = 40 hours
- Continuous replenishment
- Setup time = 8 hours
- Labor + overhead costs = $100/hr
- Unit production cost = $10
- Inventory holding cost = 20%

Demand = 50 parts per week

**EPQ- Example Continued**

\[
D \ (Demand) = 50 \ (52) = 2600 \ units \ /yr.
\]

\[
P \ (Production) = 4 \ (40) \ (52) = 8320 \ units \ /yr.
\]

- Setup Cost = 8 (100) = $800 /order
- H (holding cost) = 0.2 (10) = $2 /unit /yr.

\[
EPQ = \frac{2(800)2600}{\sqrt{2 \left(1 - \frac{2600}{8320}\right)}} = 1740 \ units \ / \ order
\]
EPQ - Example Continued

\[ tp = \frac{Q^*}{P} = \frac{1740}{160} = 10.875 \text{ weeks} \]
\[ = 10.875(40 \text{ hours}) = 435 \text{ hours} \]

\[ I_{max} = tp (P - D) = 435 (2.75) = 1196 \text{ units} \]
\[ td = \frac{I_{max}}{D} = \frac{1196}{1.25} = 957 \text{ hours}, \]
\[ T = tp + td = 435 + 957 = 1392 \text{ hours or} \]
\[ T = \frac{Q^*}{D} = \frac{1740}{1.25} = 1392 \text{ hours} \]
Model III (EOQ with backordering)

- If Items are too expensive to hold sufficient inventory for demand!
- Or there is no substitute for the company!
- Then backordering may be preferred!

**Objective:**
- Determine lot size \((Q^*)\) and min. inventory level \((B^*)\)
- so as to minimize \(TC(Q,B)\)
Model III (EOQ with backordering)

\[ t_1 = \text{time during which the demand is satisfied from inventory} \]

\[ t_2 = \text{time during which demand is backordered until the next order arrival time} \]

\[ T = \text{inventory cycle length} = t_1 + t_2 = \frac{Q}{D} \]

Inventory holding cost

\[ = H \frac{(Q - B)^2}{2Q} \]

Backordering cost

\[ = b \frac{B^2}{2Q} \]

\( b = \text{backordering cost per unit per year} \)

\[ TC(Q, B) = cD + A \frac{D}{Q} + H \frac{(Q - B)^2}{2Q} + b \frac{B^2}{2Q} \]
Model III (EOQ with backordering)

\[
\frac{dTC(Q)}{dQ} = 0 \quad \Rightarrow \quad Q^* = \sqrt{\frac{2AD}{H}} \sqrt{\frac{H + b}{b}}
\]

\[
\frac{dTC(B)}{dB} = 0 \quad \Rightarrow \quad B^* = \sqrt{\frac{2AD}{b}} \sqrt{\frac{H}{H + b}} = \frac{HQ^*}{H + b}
\]

EOQ with backordering-Example

Suppose plant should work in its full capacity for 52 weeks in one year.

- Lead time = 2 weeks
- Unlimited capacity or fixed lead time
- \(c = \$500\) per unit
- \(i = 20\%\)
- Production rate = 4 parts/hr
- Available hours per week = 40 hours
- \(A = \$1000\) (fixed cost per order)
- \(b = \$75\) per unit per year

Determine the lot size and min. inventory level of inventory system.
Example Continued

Economic Order Quantity:

\[ Q^* = \sqrt{\frac{2AD}{H}} \sqrt{\frac{H + b}{b}} \quad \Rightarrow \quad EOQ = \sqrt{\frac{2(1000)(8320)}{100}} \sqrt{\frac{175}{75}} = 623 \text{ units} \]

Each inventory period length = \( Q^* / D = 623 / 8320 \) = 0.075 years = 3.89 weeks = 156 hours

Example Continued

Maximum Backorder Level:

\[ B^* = \frac{HQ^*}{H + b} \quad \Rightarrow \quad B^* = \frac{100(623)}{175} = 356 \text{ units} \]

Minimum inventory level = – 356
Example Continued

- Annual ordering cost = $13355
- Annual holding cost = $5721
- \( D/Q \) (number of orders) = 13.355
- Annual backordering cost = $7629

Example Continued

\[ t1 = \frac{(Q^* - B^*)}{D} = \frac{267}{160} = 1.67 \text{ weeks} \]
\[ t2 = \frac{B^*}{D} = \frac{356}{160} = 2.22 \text{ weeks} \]

\[ T = t1 + t2 = 1.67 + 2.22 = 3.89 \text{ weeks} \]

or

\[ T = \frac{Q^*}{D} = \frac{623}{160} = 3.89 \text{ weeks} \]
Example Continued

Model I vs. Model III

Model III (EOQ with backordering):
EOQ = 623 units and B* = 356 units

\[
TC(EOQ, B^*) = \frac{(623 - 356)^2}{2(623)} H + \frac{(356)^2}{2(623)} b + A \frac{8320}{623}
\]

H = $100, b = $75, A = $1000,

excluding Purchasing cost of (cD) = 500(8320) = $4,160,000

\[
TC(EOQ, B^*) = 5721 + 7629 + 13355 = 26705
\]
Model I vs. Model III

Model I (EOQ without backordering):
EOQ = 408 units

$$TC(EOQ) = \frac{408}{2} H + A \frac{8320}{408}$$

$$TC(EOQ) = 20400 + 20400 = 40800$$

$TC$ of Model III < $TC$ of Model I

Model IV (EPQ with Backordering)
Model IV (EPQ with Backordering)

Inventory holding cost = \[ H \frac{Q \left( 1 - \frac{D}{P} \right) - B}{2Q \left( 1 - \frac{D}{P} \right)} \]

Backordering cost = \[ b \frac{B^2}{2Q \left( 1 - \frac{D}{P} \right)} \]

Total Inventory Cost

\[ TC(Q, B) = cD + S \frac{D}{Q} + H \frac{Q \left( 1 - \frac{D}{P} \right) - B}{2Q \left( 1 - \frac{D}{P} \right)} + b \frac{B^2}{2Q \left( 1 - \frac{D}{P} \right)} \]
EPQ with Backordering

\[
\frac{dTC(Q)}{dQ} = 0 \quad \Rightarrow \quad Q^* = \sqrt{\frac{2SD}{H(1 - \frac{D}{P})}} \sqrt{\frac{H + b}{b}}
\]

\[
\frac{dTC(B)}{dB} = 0 \quad \Rightarrow \quad B^* = \frac{HQ * \left(1 - \frac{D}{P}\right)}{H + b}
\]

EPQ with backordering-Example

Suppose for a certain product type you need to produce weekly uniform demand as stated below:

- Production rate = 4 parts /hr
- Available hours per week = 40 hours
- Continuous replenishment
- Setup time = 8 hours
- Labor + overhead costs = $100 /hr
- Unit production cost = $10
- Inventory holding cost = 20%

Demand = 50 parts per week
b = $100 per unit per year
Example Continued

\[ D \text{ (Demand)} = 50 \times 52 = 2600 \text{ units/yr.} \]
\[ P \text{ (Production)} = 4 \times 40 \times 52 = 8320 \text{ units/yr.} \]
\[ \text{Setup Cost} = 8 \times 100 = 800 \text{ /order} \]
\[ H \text{ (holding cost)} = 0.2 \times 10 = 2 \text{ /unit/yr.} \]

\[ EPQ = \sqrt{\frac{2(800)2600}{102}} = \sqrt{1757} \text{ units/order} \]

\[ B^* = \frac{HQ^*\left(1 - \frac{D}{P}\right)}{H + b} = \frac{2(1757)(0.6875)}{102} = 24 \text{ units} \]
Example Continued

\[ t_1 = \frac{24}{2.75} = 9 \text{ hours} \]
\[ t_p = \frac{1757}{4} = 439 \text{ hours} \]
\[ I_{\text{max}} = 439(2.75) - 24 = 1183 \text{ units} \]
\[ t_2 = \frac{1183}{2.75} = 430 \text{ hours} \]
\[ t_3 = \frac{1183}{1.25} = 947 \text{ hours} \]
\[ t_4 = \frac{24}{1.25} = 19 \text{ hours} \]
\[ t_d = t_3 + t_4 = 966 \text{ hours} \]
\[ T = t_p + t_d = 439 + 966 = 1405 \text{ hours or} \]
\[ T = \frac{1757}{1.25} = 1405 \text{ hours} \]
Example Continued

Average inventory = \[
\frac{1757 \left(1 - \frac{2600}{8320}\right) - 24}{2(1757) \left(1 - \frac{2600}{8320}\right)} = 580 \text{ units}
\]

Average Backorder = \[
\frac{24^2}{2(1757) \left(1 - \frac{2600}{8320}\right)} = 0.24 \text{ units}
\]